HEXAFLEXAGONS, PROBABILITY PARADOXES, AND THE TOWER OF HANOI

For 25 of his 90 years, Martin Gardner wrote "Mathematical Games and Recreations," a monthly column for *Scientific American* magazine. These columns have inspired hundreds of thousands of readers to delve more deeply into the large world of mathematics. He has also made significant contributions to magic, philosophy, debunking pseudoscience, and children's literature. He has produced more than 60 books, including many best sellers, most of which are still in print. His *Annotated Alice* has sold more than a million copies. He continues to write a regular column for the *Skeptical Inquirer* magazine. (The photograph is of the author at the time of the first edition.)

THE NEW MARTIN GARDNER MATHEMATICAL LIBRARY

From 1957 through 1986 Martin Gardner wrote the "Mathematical Games" columns for *Scientific American* that are the basis for these books. *Scientific American* editor Dennis Flanagan noted that this column contributed substantially to the success of the magazine. The exchanges between Martin Gardner and his readers gave life to these columns and books. These exchanges have continued and the impact of the columns and books has grown. These new editions give Martin Gardner the chance to bring readers up to date on newer twists on old puzzles and games, on new explanations and proofs, and on links to recent developments and discoveries. Illustrations have been added and existing ones improved, and the bibliographies have been greatly expanded throughout.

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Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi

MARTIN GARDNER'S FIRST BOOK OF MATHEMATICAL PUZZLES AND GAMES

Martin Gardner

The Mathematical Association of America

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521756150

First published 2008

First edition published as The SCIENTIFIC AMERICAN Book of Mathematical Puzzles & Diversions, Simon and Schuster, 1959

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Gardner, Martin, 1914–
Hexaflexagons, probability paradoxes, and the Tower of Hanoi : Martin Gardner's first book of mathematical puzzles and games / Martin Gardner.
p. cm. – (The new Martin Gardner mathematical library)
Includes bibliographical references and index.
ISBN 978-0-521-75615-0 (hardback)
1. Mathematical recreations. I. Title. II. Series.
QA95.G247 2008
793.74 – dc22 2008012533

ISBN 978-0-521-75615-0 hardback
ISBN 978-0-521-73525-4 paperback

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Acknowledgments

MARTIN GARDNER thanks Scientific American for allowing reuse of material from his columns in that magazine, material Copyright © 1956 (Chapter 1), and 1957 (Chapters 2–13), and 1958 (Chapters 14–16) by Scientific American, Inc. He also thanks the artists who contributed to the success of these columns and books for allowing reuse of their work: James D. Egelson (via heirs Jan and Nicholas Egleson), Irving Geis (via heir Sandy Geis), Harold Jacobs, Amy Kasai, and Bunji Tagawa (via Donald Garber for the Tagawa Estate). Artists names are cited where these were known. All rights other than use in connection with these materials lie with the original artists.

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Introduction to the First Edition

THE ELEMENT OF PLAY, which makes recreational mathematics recreational, may take many forms: a puzzle to be solved, a competitive game, a magic trick, paradox, fallacy, or simply mathematics with any sort of curious or amusing fillip. Are these examples of pure or applied mathematics? It is hard to say. In one sense recreational mathematics is pure mathematics, uncontaminated by utility. In another sense it is applied mathematics, for it meets the universal human need for play.

Perhaps this need for play is behind even pure mathematics. There is not much difference between the delight a novice experiences in cracking a clever brain teaser and the delight a mathematician experiences in mastering a more advanced problem. Both look on beauty bare – that clean, sharply defined, mysterious, entrancing order that underlies all structure. It is not surprising, therefore, that it is often difficult to distinguish pure from recreational mathematics. The four-color map theorem, for example, is an important theorem in topology, yet discussions of the theorem will be found in many recreational volumes. No one can deny that paper flexagons, the subject of this book's opening chapter, are enormously entertaining toys; yet an analysis of their structure takes one quickly into advanced group theory, and articles on flexagons have appeared in technical mathematical journals.

Creative mathematicians are seldom ashamed of their interest in recreational mathematics. Topology had its origin in Euler's analysis of a puzzle about crossing bridges. Leibniz devoted considerable time to the study of a peg-jumping puzzle that recently enjoyed its latest revival under the trade name of Test Your High-Q. David

Hilbert, the great German mathematician, proved one of the basic theorems in the field of dissection puzzles. Alan Turing, a pioneer in modern computer theory, discussed Sam Loyd's 15-puzzle (here described in Chapter 9) in an article on solvable and unsolvable problems. I have been told by Piet Hein (whose game of Hex is the subject of Chapter 8) that when he visited Albert Einstein he found a section of Einstein's bookshelf devoted to books on recreational mathematics. The interest of those great minds in mathematical play is not hard to understand, for the creative thought bestowed on such trivial topics is of a piece with the type of thinking that leads to mathematical and scientific discovery. What is mathematics, after all, except the solving of puzzles? And what is science if it is not a systematic effort to get better and better answers to puzzles posed by nature?

The pedagogic value of recreational mathematics is now widely recognized. One finds an increasing emphasis on it in magazines published for mathematics teachers, and in the newer textbooks, especially those written from the "modern" point of view. *Introduction to Finite Mathematics*, for example, by J. G. Kemeny, J. Laurie Snell, and Gerald L. Thompson, is livened by much recreational material. These items hook a student's interest as little else can. The high school mathematics teacher who reprimands two students for playing a surreptitious game of ticktacktoe instead of listening to the lecture might well pause and ask: "Is this game more interesting mathematically to these students than what I am telling them?" In fact, a classroom discussion of ticktacktoe is not a bad introduction to several branches of modern mathematics.

In an article on "The Psychology of Puzzle Crazes" (*Nineteenth Century Magazine*, December 1926) the great English puzzlist Henry Ernest Dudeney made two complaints. The literature of recreational mathematics, he said, is enormously repetitious, and the lack of an adequate bibliography forces enthusiasts to waste time in devising problems that have been devised long before. I am happy to report that the need for such a bibliography has at last been met. Professor William L. Schaaf, of Brooklyn College, compiled an excellent four-volume bibliography, titled *Recreational Mathematics*, which can be obtained from the National Council of Teachers of Mathematics. As to Dudeney's other complaint, I fear that it still applies to current

books in the field, including this one, but I think readers will discover here more than the usual portion of fresh material that has not previously found its way between book covers.

I would like to thank Gerard Piel, publisher of *Scientific American*, and Dennis Flanagan, editor, for the privilege of appearing regularly in the distinguished company of their contributors, and for permission to reprint my efforts in the present volume. And I am grateful also to the thousands of readers, from all parts of the world, who have taken the trouble to call my attention to mistakes (alas too frequent) and to make valuable suggestions. In some cases this welcome feedback has been incorporated into the articles themselves, but in most cases it is pulled together in an addendum at the end of each chapter. The answers to problems (where necessary) also appear at the end of the chapter. A bibliography of selected references for further reading will be found at the close of the book.

And I must not fail to thank my wife, not only for competent and fairly cheerful proofreading, but also for her patience during those trying moments of mathematical meditation when I do not hear what she is saying.

Martin Gardner
Dobbs Ferry, New York, 1959

Preface to the Second Edition

FOR MORE THAN TWENTY-FIVE YEARS I wrote a monthly column on recreational mathematics for *Scientific American*. Those columns have been reprinted in fifteen books. In 2005, when the Mathematical Association of America (MAA) put all fifteen on a CD, type was not reset. This severely limited what I could add to update the columns and expand bibliographies. Because Cambridge University Press is resetting type, I am now happily free to add as much fresh material as I please.

I am indebted to Don Albers and to Peter Renz for initiating the MAA's joint venture with Cambridge to produce a uniform set of all the *Scientific American* books and to Elwyn Berlekamp for support of preparation of the manuscript. I am equally indebted to my many readers, both professional and amateur mathematicians, for supplying so much new material for my columns.

I'm not a creative mathematician. I am a journalist who loves math and who enjoys writing about what the real mathematicians discover. Note that I say "discover." I'm an unabashed Platonist who believes, with all the great mathematicians past and present, that mathematical truth is independent of human cultures. It is as firmly "out there," in its own strange and mysterious abstract realm, as the stars are out there as material structures not made by us.

Martin Gardner
Norman, Oklahoma, 2008

Tower of Hanoi (which also goes by other names like Tower of Brahma or The Lucas Tower) is a recreational mathematical puzzle by the Edouard Lucas in the year 1883.Â Famous examples of these are the Conwayâ€™s Game of life (a cellular automaton), the Monty Hall problem (a probability puzzle, which is characteristically also similar to a veridical paradox), Bertrandâ€™s Box problem and Fractals. The Tower of Hanoi is one such mathematical game. It is also known as the Tower of Brahma or the Lucas Tower. How, when, and where did it all begin. This puzzle is of unconfirmed origins, with many legends presenting different stories about where it came from. Please Update (Trackers Info) Before Start "Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi - Martin Gardner [PDF] - [Sherlockism]" Torrent Downloading to See Updated Seeders And Leechers for Batter Torrent Download Speed.Â Torrent File Content (1 file). Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi - Martin Gardner Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi - Martin Gardner.pdf