

PARTICLES AND QUANTUM FIELDS

Particles and Quantum Fields

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*To my wife Annemarie
and our son Hagen Michael II*

Preface

This book arose from lectures I gave at the Freie Universität Berlin over the past five decades. They were intended to prepare graduate students for their research in elementary-particle physics or in many-body theory of condensed matter. They should serve as a general introduction and a basis for understanding more advanced work on the subject.

The theory of quantum fields presented in this book is mainly based on the perturbative approach. Elementary particles are introduced initially without any interactions. These are added later, and their strength is parametrized by some coupling constant g . The consequences are studied order by order in g , with the particles propagating forward from interaction to interaction. Such a treatment is clearly a gross simplification of what happens in nature, where even the existence of a free particle involves the full interaction from the very beginning. Nevertheless, this kind of procedure has been the basis of many successful theories. In all of them, there exist dominant freely propagating excitations or elementary particles at least in some experimentally accessible limit. The most prominent example is the theory of strongly interacting particles. There they are described as being composed of quarks held together by gluons which interact via a nonabelian gauge theory called quantum chromodynamics (QCD). In the limit of large energies, the particles behave like free point-like particles. This behavior was named *parton-like* by Richard Feynman. The existence of such a limiting behavior in QCD was called *asymptotic freedom*. It was the main reason for the possibility of developing a theory for these particles, which gave good explanations of many interaction processes between elementary particles. The initial *creation* of the particles, however, remained far from being understood. It involves a regime of strong interactions where perturbation theory fails.

A field-theoretic method to reach into this regime has been developed in quantum field theory of many-body physics. There a separation of the two regimes, the formation of particle-like excitation and their scattering, is much clearer to observe experimentally than in particle physics. For this reason, many-body theory has been a major source of inspiration for the development of theoretical methods to describe strongly interacting phenomena. An extension of perturbation theory into the strongly interacting regime has so far been possible mainly by employing resummation techniques. Initially, theorists have summed certain classes of Feynman diagrams by repeating infinitely many times the same interaction leading to a geometric series whose sum can be evaluated. This has allowed the understanding of many pronounced observable phenomena as consequences of a sum of infinitely

many bubbles and ladders of diagrams. The methods for this were developed by Hartree, Fock, and Bogoliubov in many-body theory, and by Bethe and Salpeter in quantum electrodynamics.

The development of renormalization group theory has led to a generalization of this method. It permits to extend the sum of bubbles and ladders to sums of diagrams of many different topologies. This makes them applicable in the regime of strong couplings, where they can be used to study various many-body phenomena even in the so-called *critical regime*. There the interactions become so strong that they are much more important than the free-particle propagation.

In many-body theory, one can parametrize the separation of the two regimes quite clearly by formulating the theory on a lattice. The propagation is characterized by a so-called hopping amplitude from lattice point to lattice point. The critical regime is reached when the masses of some of the participating excitations go to zero. In this limit, the range of their propagation tends to infinity, and their interaction becomes increasingly important.

An efficient alternative to the summation of infinitely many perturbation-theoretic diagrams is based on a variational approach. Its power was discovered in 1877 by John Rayleigh and formalized by Walter Ritz in 1908. Some time ago, the theory was revived by Feynman and Kleinert.¹ They set up a first-order variational approximation to path integrals, which led to reasonable approximations for a variety of quantum mechanical problems. The approximations were later expanded to all orders, and have finally led to the powerful *field-theoretic variational perturbation theory* (VPT). In that form, the theory is able to simplify and replace the popular renormalization group approach of critical phenomena. It has been successfully applied to many phase transitions, and is published in a monograph.²

An important aspect of a theory of critical phenomena is the fact that the free-field propagators play no longer the important role they have in perturbation expansions. The underlying free-particle behavior is based on a Gaussian approximation to field fluctuations. In the critical regime, this approximation of the distributions has tails which follow power-like distributions. Such tails are observed in the statistics of very rare events, which are called “black-swan events”.³ These occur in nature in many different circumstances, ranging from oceanic monster waves over earthquakes and wind gusts, to catastrophic crashes of financial markets.⁴

I want to thank my friend Remo Ruffini for creating an extremely lively and inspiring environment for scientific work in particle and astrophysics at many exciting places of the globe, where I was invited for lectures and discussions of topics of this

¹R.P. Feynman and H. Kleinert, Phys. Rev. A *34*, 5080 (1986).

²H. Kleinert and V. Schulte-Frohlinde, *Critical Properties of Φ^4 -Theories*, World Scientific, Singapore 2001, pp. 1–489 (<http://klnrt.de/b8>). See Chapter 20 for the variational approach.

³H. Kleinert, *Quantum Field Theory of Black-Swan Events*, EPL *100*, 10001 (2013) (www.ejtp.com/articles/ejtpv11i31p1.pdf); *Effective Action and Field Equation for BEC from Weak to Strong Couplings*, J. Phys. B *46*, 175401 (2013) (<http://klnrt.de/403>).

⁴H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, World Scientific, Singapore, 2009 (<http://klnrt.de/b5>). See Chapter 20.

book. Ruffini, who holds a chair in theoretical physics at the university of Rome “La Sapienza”, founded an international center which I am part of, where scientists from all over the world do research, and where students can prepare their Ph.D. degree (for details see ICRANet.org).

I am also very grateful to my colleague Axel Pelster who, for many years, has shared with me the burden and joy of bringing students of the Freie Universität Berlin to their master’s and doctor’s degrees. His careful reading of large parts of the manuscript has produced useful insights and corrections.

Another person who greatly helped me to spot errors in Chapters 23 and 24 is my former CERN colleague and friend Franco Buccella, professor of theoretical physics in beautiful Naples. These chapters were also proofread by Harald Fritzsche. In Chapter 30 several corrections came from Claus Kiefer, Hans Ohanian, Kellogg Stelle, Pisin Chen, She-Sheng Xue, and Václav Zatloukal.

Finally I want to thank Dr. Annemarie Kleinert for her patience and sacrifice of a lot of time, in which she set aside her own research projects to help me spot and correct many errors. Without her repeated reading the manuscript and her persistent encouragement, the book would certainly never have been finished.

The reader who detects errors, is kindly asked to report them by email to h@klrnt.de.

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Berlin, April 2016

Contents

Preface	vii
1 Fundamentals	1
1.1 Classical Mechanics	1
1.2 Relativistic Mechanics in Curved Spacetime	10
1.3 Quantum Mechanics	11
1.3.1 Bragg Reflections and Interference	11
1.3.2 Matter Waves	13
1.3.3 Schrödinger Equation	14
1.3.4 Particle Current Conservation	17
1.4 Dirac's Bra-Ket Formalism	18
1.4.1 Basis Transformations	19
1.4.2 Bracket Notation	20
1.4.3 Continuum Limit	22
1.4.4 Generalized Functions	24
1.4.5 Schrödinger Equation in Dirac Notation	25
1.4.6 Momentum States	27
1.4.7 Incompleteness and Poisson's Summation Formula	29
1.5 Observables	31
1.5.1 Uncertainty Relation	32
1.5.2 Density Matrix and Wigner Function	33
1.5.3 Generalization to Many Particles	34
1.6 Time Evolution Operator. Definition	35
1.7 Time Evolution Operator. Properties	38
1.8 Heisenberg Picture of Quantum Mechanics	40
1.9 Interaction Picture and Perturbation Expansion	43
1.10 Time Evolution Amplitude	44
1.11 Fixed-Energy Amplitude	47
1.12 Free-Particle Amplitudes	49
1.13 Quantum Mechanics of General Lagrangian Systems	53
1.14 Particle on the Surface of a Sphere	58
1.15 Spinning Top	61
1.16 Classical and Quantum Statistics	69
1.16.1 Canonical Ensemble	69
1.16.2 Grand-Canonical Ensemble	70
1.17 Density of States and Tracelog	75

Appendix 1A	Simple Time Evolution Operator	77
Appendix 1B	Convergence of the Fresnel Integral	77
Appendix 1C	The Asymmetric Top	78
	Notes and References	80
2	Field Formulation of Many-Body Quantum Physics	82
2.1	Mechanics and Quantum Mechanics for n Nonrelativistic Particles	82
2.2	Identical Particles: Bosons and Fermions	85
2.3	Creation and Annihilation Operators for Bosons	91
2.4	Schrödinger Equation for Noninteracting Bosons in Terms of Field Operators	95
2.5	Second Quantization and Symmetrized Product Representation	97
2.6	Bosons with Two-Body Interactions	101
2.7	Quantum Field Formulation of Many-Body Schrödinger Equations for Bosons	102
2.8	Canonical Formalism in Quantum Field Theory	104
2.9	More General Creation and Annihilation Operators	109
2.10	Quantum Field Formulation of Many-Fermion Schrödinger Equations	111
2.11	Free Nonrelativistic Particles and Fields	113
2.12	Second-Quantized Current Conservation Law	116
2.13	Free-Particle Propagator	117
2.14	Collapse of Wave Function	120
2.15	Quantum Statistics of Free Nonrelativistic Fields	121
	2.15.1 Thermodynamic Quantities	121
	2.15.2 Degenerate Fermi Gas Near $T = 0$	127
	2.15.3 Degenerate Bose Gas Near $T = 0$	132
	2.15.4 High Temperatures	137
2.16	Noninteracting Bose Gas in a Trap	138
	2.16.1 Bose Gas in a Finite Box	138
	2.16.2 Harmonic and General Power Trap	141
	2.16.3 Anharmonic Trap in Rotating Bose-Einstein Gas	142
2.17	Temperature Green Functions of Free Particles	143
2.18	Calculating the Matsubara Sum via Poisson Formula	148
2.19	Nonequilibrium Quantum Statistics	150
	2.19.1 Linear Response and Time-Dependent Green Functions for $T \neq 0$	150
	2.19.2 Spectral Representations of Green Functions for $T \neq 0$	153
2.20	Other Important Green Functions	156
2.21	Hermitian Adjoint Operators	159
2.22	Harmonic Oscillator Green Functions for $T \neq 0$	160
	2.22.1 Creation Annihilation Operators	160
	2.22.2 Real Field Operators	163
Appendix 2A	Permutation Group and Representations	165
Appendix 2B	Treatment of Singularities in Zeta-Function	169

2B.1	Finite Box	170
2B.2	Harmonic Trap	172
	Notes and References	174
3	Interacting Nonrelativistic Particles	177
3.1	Weakly Interacting Bose Gas	178
3.2	Weakly Interacting Fermi Gas	188
3.2.1	Electrons in a Metal	188
3.3	Superconducting Electrons	196
3.3.1	Zero Temperature	201
3.4	Renormalized Theory at Strong Interactions	205
3.4.1	Finite Temperature	207
3.5	Crossover to Strong Couplings	211
3.5.1	Bogoliubov Theory for Bose Gas at Finite Temperature	212
3.6	Bose Gas at Strong Interactions	214
3.7	Corrections Due to Omitted Interaction Hamiltonian	231
	Appendix 3A Two-Loop Momentum Integrals	234
	Notes and References	237
4	Free Relativistic Particles and Fields	240
4.1	Relativistic Particles	240
4.2	Differential Operators for Lorentz Transformations	247
4.3	Space Inversion and Time Reversal	257
4.4	Free Relativistic Scalar Fields	258
4.5	Other Symmetries of Scalar Action	265
4.5.1	Translations of Scalar Field	266
4.5.2	Space Inversion of Scalar Field	267
4.5.3	Time Reversal of Scalar Field	268
4.5.4	Charge Conjugation of Scalar Field	272
4.6	Electromagnetic Field	272
4.6.1	Action and Field Equations	273
4.6.2	Gauge Invariance	275
4.6.3	Lorentz Transformation Properties of Electromagnetic Fields	278
4.7	Other Symmetries of Electromagnetic Action	280
4.7.1	Translations of the Vector Field	281
4.7.2	Space Inversion, Time Reversal, and Charge Conjugation of the Vector Field	281
4.8	Plane-Wave Solutions of Maxwell's Equations	282
4.9	Gravitational Field	287
4.9.1	Action and Field Equations	288
4.9.2	Lorentz Transformation Properties of Gravitational Field	291
4.9.3	Other Symmetries of Gravitational Action	292
4.9.4	Translations of Gravitational Field	292

4.9.5	Space Inversion, Time Reversal, and Charge Conjugation of Gravitational Field	292
4.9.6	Gravitational Plane Waves	293
4.10	Free Relativistic Fermi Fields	299
4.11	Spin-1/2 Fields	300
4.12	Other Symmetries of Dirac Action	310
4.12.1	Translations and Poincaré Group	310
4.12.2	Space Inversion	310
4.12.3	Dirac's Original Derivation	316
4.12.4	Maxwell Equations Written à la Dirac	318
4.12.5	Pauli-Villars Equation for the Klein-Gordon Field	320
4.12.6	Charge Conjugation	320
4.12.7	Time Reversal	323
4.12.8	Transformation Properties of Currents	325
4.13	Majorana Fields	326
4.13.1	Plane-Wave Solutions of Dirac Equation	329
4.14	Lorentz Transformation of Spinors	340
4.15	Precession	343
4.15.1	Wigner Precession	343
4.15.2	Thomas Precession	344
4.15.3	Spin Four-Vector and Little Group	345
4.16	Weyl Spinor Calculus	348
4.17	Massive Vector Fields	350
4.17.1	Action and Field Equations	350
4.17.2	Plane Wave Solutions for Massive Vector Fields	351
4.18	Higher-Spin Representations	354
4.18.1	Rotations	354
4.18.2	Extension to Lorentz Group	357
4.18.3	Finite Representation Matrices	359
4.19	Higher Spin Fields	365
4.19.1	Plane-Wave Solutions	368
4.20	Vector Field as a Higher-Spin Field	369
4.21	Rarita-Schwinger Field for Spin 3/2	370
Appendix 4A	Derivation of Baker-Campbell-Hausdorff Formula	371
Appendix 4B	Wigner Rotations and Thomas Precession	373
Appendix 4C	Calculation in Four-Dimensional Representation	376
Appendix 4D	Hyperbolic Geometry	377
Appendix 4E	Clebsch-Gordan Coefficients	379
Appendix 4F	Spherical Harmonics	383
Appendix 4G	Projection Matrices for Symmetric Tensor Fields	385
Notes and References	386

5	Classical Radiation	389
5.1	Classical Electromagnetic Waves	389
5.1.1	Electromagnetic Field of a Moving Charge	390
5.1.2	Dipole Moment	395
5.2	Classical Gravitational Waves	396
5.2.1	Gravitational Field of Matter Source	396
5.2.2	Quadrupole Moment	402
5.2.3	Average Radiated Energy	405
5.3	Simple Models for Sources of Gravitational Radiation	406
5.3.1	Vibrating Quadrupole	406
5.3.2	Two Rotating Masses	408
5.3.3	Particle Falling into Star	414
5.3.4	Cloud of Colliding Stars	417
5.4	Orders of Magnitude of Different Radiation Sources	418
5.5	Detection of Gravitational Waves	420
5.6	Inspiralling Plunge of One Black Hole into another	423
	Appendix 5A Attractive Gravity versus Repulsive Electromagnetism	424
	Appendix 5B Nonlinear Gravitational Waves	424
	Appendix 5C Nonexistence of Gravitational Waves in $D = 3$ and $D = 2$	426
	Appendix 5D Precession of Gyroscope in a Satellite Orbit	430
	Notes and References	434
6	Relativistic Particles and Fields in External Electromagnetic Potential	436
6.1	Charged Point Particles	436
6.1.1	Coupling to Electromagnetism	437
6.1.2	Spin Precession in an Atom	439
6.1.3	Relativistic Equation of Motion for Spin Vector and Thomas Precession	442
6.2	Charged Particle in Schrödinger Theory	445
6.3	Charged Relativistic Fields	447
6.3.1	Scalar Field	447
6.3.2	Dirac Field	448
6.4	Pauli Equation from Dirac Theory	449
6.5	Relativistic Wave Equations in the Coulomb Potential	451
6.5.1	Reminder of the Schrödinger Equation in a Coulomb Potential	452
6.5.2	Klein-Gordon Field in a Coulomb Potential	454
6.5.3	Dirac Field in a Coulomb Potential	455
6.6	Green Function in an External Electromagnetic Field	461
6.6.1	Scalar Field in a Constant Electromagnetic Field	461
6.6.2	Dirac Field in a Constant Electromagnetic Field	467
6.6.3	Dirac Field in an Electromagnetic Plane-Wave Field	469
	Appendix 6A Spinor Spherical Harmonics	472
	Notes and References	473

7	Quantization of Relativistic Free Fields	474
7.1	Scalar Fields	475
7.1.1	Real Case	475
7.1.2	Field Quantization	475
7.1.3	Propagator of Free Scalar Particles	481
7.1.4	Complex Case	485
7.1.5	Energy of Free Charged Scalar Particles	487
7.1.6	Behavior under Discrete Symmetries	488
7.2	Spacetime Behavior of Propagators	494
7.2.1	Wick Rotation	495
7.2.2	Feynman Propagator in Minkowski Space	497
7.2.3	Retarded and Advanced Propagators	500
7.2.4	Comparison of Singular Functions	504
7.3	Collapse of Relativistic Wave Function	508
7.4	Free Dirac Field	509
7.4.1	Field Quantization	509
7.4.2	Energy of Free Dirac Particles	512
7.4.3	Lorentz Transformation Properties of Particle States	514
7.4.4	Behavior under Discrete Symmetries	523
7.5	Free Photon Field	527
7.5.1	Field Quantization	528
7.5.2	Covariant Field Quantization	533
7.5.3	Gupta-Bleuler Subsidiary Condition	552
7.5.4	Behavior under Discrete Symmetries	559
7.6	Massive Vector Bosons	560
7.6.1	Field Quantization	561
7.6.2	Energy of Massive Vector Particles	563
7.6.3	Propagator of Massive Vector Particles	564
7.7	Wigner Rotation of Spin-1 Polarization Vectors	568
7.7.1	Behavior under Discrete Symmetry Transformations	570
7.8	Spin-3/2 Fields	571
7.9	Gravitons	573
7.10	Spin-Statistics Theorem	574
7.11	CPT-Theorem	578
7.12	Physical Consequences of Vacuum Fluctuations. Casimir Effect	578
7.13	Zeta Function Regularization	585
7.14	Dimensional Regularization	588
7.15	Accelerated Frame and Unruh Temperature	591
7.16	Photon Propagator in Dirac Quantization Scheme	593
7.17	Free Green Functions of n Fields	594
7.17.1	Wick's Theorem	598
7.18	Functional Form of Wick's Theorem	602
7.18.1	Thermodynamic Version of Wick's Theorem	606
Appendix 7A	Euler-Maclaurin Formula	610

Appendix 7B Liénard-Wiechert Potential	614
Appendix 7C Equal-Time Commutator from Time-Ordered Products . . .	615
Notes and References	617
8 Continuous Symmetries and Conservation Laws.	
Noether's Theorem	619
8.1 Point Mechanics	619
8.1.1 Continuous Symmetries and Conservation Law	619
8.1.2 Alternative Derivation	621
8.2 Displacement and Energy Conservation	622
8.3 Momentum and Angular Momentum	624
8.3.1 Translational Invariance in Space	624
8.3.2 Rotational Invariance	625
8.3.3 Center-of-Mass Theorem	626
8.3.4 Conservation Laws Resulting from Lorentz Invariance	628
8.4 Generating the Symmetry Transformations	630
8.5 Field Theory	632
8.5.1 Continuous Symmetry and Conserved Currents	632
8.5.2 Alternative Derivation	633
8.5.3 Local Symmetries	634
8.6 Canonical Energy-Momentum Tensor	636
8.6.1 Electromagnetism	637
8.6.2 Dirac Field	638
8.7 Angular Momentum	640
8.8 Four-Dimensional Angular Momentum	641
8.9 Spin Current	643
8.9.1 Electromagnetic Fields	643
8.9.2 Dirac Field	646
8.10 Symmetric Energy-Momentum Tensor	648
8.10.1 Gravitational Field	650
8.11 Internal Symmetries	651
8.11.1 U(1)-Symmetry and Charge Conservation	651
8.11.2 SU(N)-Symmetry	652
8.11.3 Broken Internal Symmetries	653
8.12 Generating the Symmetry Transformations of Quantum Fields . . .	653
8.13 Energy Momentum Tensor of a Relativistic Massive Point Particle .	655
8.14 Energy Momentum Tensor of a Massive Charged Particle in a Maxwell Field	656
Notes and References	659
9 Scattering and Decay of Particles	660
9.1 Quantum-Mechanical Description	660
9.1.1 Schrödinger Picture	660
9.1.2 Heisenberg Picture	661

9.1.3	Interaction Picture	662
9.1.4	Neumann-Liouville Expansion	662
9.1.5	Møller Operators	664
9.1.6	Lippmann-Schwinger Equation	667
9.1.7	Discrete States	669
9.1.8	Gell-Mann-Low Formulas	670
9.2	Scattering by External Potential	675
9.2.1	The T -Matrix	675
9.2.2	Asymptotic Behavior	679
9.2.3	Partial Waves	681
9.2.4	Off Shell T -Matrix	687
9.2.5	Cross Section	690
9.2.6	Partial Wave Decomposition of Total Cross Section	694
9.2.7	Dirac δ -Function Potential	695
9.2.8	Spherical Square-Well Potential	697
9.3	Two-Particle Scattering	700
9.3.1	Center-of-Mass Scattering Cross Section	701
9.3.2	Laboratory Scattering Cross Section	703
9.4	Decay	707
9.5	Optical Theorem	707
9.6	Initial- and Final-State Interactions	708
9.7	Tests of Time-Reversal Violations	709
9.7.1	Strong and Electromagnetic Interactions	710
9.7.2	Selection Rules in Weak Interactions	711
9.7.3	Phase of Weak Amplitudes from Time-Reversal Invariance	712
Appendix 9A	Green Function in Arbitrary Dimensions	713
Appendix 9B	Partial Waves in Arbitrary Dimensions	715
Appendix 9C	Spherical Square-Well Potential in D Dimensions	720
	Notes and References	722
10	Quantum Field Theoretic Perturbation Theory	723
10.1	The Interacting n -Point Function	723
10.2	Perturbation Expansion for Green Functions	725
10.3	Feynman Rules for ϕ^4 -Theory	727
10.3.1	The Vacuum Graphs	729
10.4	The Two-Point Function	732
10.5	The Four-Point Function	734
10.6	Connected Green Functions	736
10.6.1	One-Particle Irreducible Graphs	740
10.6.2	Momentum Space Version of Diagrams	742
10.7	Green Functions and Scattering Amplitudes	744
10.8	Wick Rules for Scattering Amplitudes	751
10.9	Thermal Perturbation Theory	752
	Notes and References	755

11	Extracting Finite Results from Perturbation Series. Regularization, Renormalization	757
11.1	Vacuum Diagrams	757
11.2	Two- and Four-Point Functions	760
11.3	Divergences, Cutoff, and Counterterms	762
11.4	Bare Theory and Multiplicative Renormalization	769
11.5	Dimensional Regularization of Integrals	773
11.6	Renormalization of Amplitudes	787
11.7	Additive Renormalization of Vacuum Energy	790
11.8	Generalization to $O(N)$ -Symmetric Models	791
11.9	Finite S -Matrix Elements	796
	Appendix 11A Second Proof of Veltman's Integral Rule	798
	Notes and References	799
12	Quantum Electrodynamics	801
12.1	Gauge Invariant Interacting Theory	801
12.1.1	Reminder of Classical Electrodynamics of Point Particles	802
12.1.2	Electrodynamics and Quantum Mechanics	804
12.1.3	Principle of Nonholonomic Gauge Invariance	806
12.1.4	Electrodynamics and Relativistic Quantum Mechanics	807
12.2	Noether's Theorem and Gauge Fields	808
12.3	Quantization	810
12.4	Perturbation Theory	813
12.5	Ward-Takahashi Identity	818
12.6	Magnetic Moment of Electron	819
12.7	Decay of Atomic State	823
12.8	Rutherford Scattering	827
12.8.1	Classical Cross Section	827
12.8.2	Quantum-Mechanical Born Approximation	829
12.8.3	Relativistic Born Approximation: Mott Formula	829
12.9	Compton Scattering	833
12.9.1	Classical Result	834
12.9.2	Klein-Nishina Formula	835
12.10	Electron-Positron Annihilation	840
12.11	Positronium Decay	845
12.12	Bremsstrahlung	846
12.12.1	Classical Bremsstrahlung	846
12.12.2	Bremsstrahlung in Mott Scattering	849
12.13	Electron-Electron Scattering	852
12.14	Electron-Positron Scattering	854
12.15	Anomalous Magnetic Moment of Electron and Muon	857
12.15.1	Form Factors	862
12.15.2	Charge Radius	863
12.15.3	Anomalous Magnetic Moment	864

12.16	Vacuum Polarization	868
12.17	Dimensional Regularization	872
12.18	Two-Dimensional QED	873
12.19	Self-Energy of Electron	874
12.20	Ward-Takahashi Identity	877
12.21	Lamb Shift	879
12.21.1	Rough Estimate of the Effect of Vacuum Fluctuations	880
12.21.2	Relativistic Estimate	882
12.21.3	Effect of Wave Functions	883
12.21.4	Effect of the Anomalous Magnetic Moment	891
Appendix 12A	Calculation of the Dirac Trace in Klein-Nishina Formula	894
Notes and References	897
13	Formal Properties of Perturbation Theory	900
13.1	Connectedness Structure of Feynman Diagrams	900
13.2	Functional Differential Equations	901
13.3	Decomposition of Green Functions into Connected Green Functions	903
13.4	Functional Differential Equation for $W[j]$	905
13.5	Iterative Solution	905
13.6	Vertex Functions	907
13.7	The Generating Functional for Vertex Functions	907
13.8	Functional Differential Equation for $\Gamma[\Phi]$	912
13.9	Effective Action as Basis for Variational Calculations	916
13.10	Effective Potential	916
13.11	Higher Effective Actions	917
13.12	High Orders in a Simple Model	922
Notes and References	925
14	Functional-Integral Representation of Quantum Field Theory	926
14.1	Functional Fourier Transformations	926
14.2	Gaussian Functional Integral	928
14.3	Functional Formulation for Free Quantum Fields	930
14.4	Interactions	933
14.5	Euclidean Quantum Field Theory	936
14.6	Functional Integral Representation for Fermions	937
14.7	Relation Between $Z[j]$ and the Partition Function	941
14.8	Bosons and Fermions in a Single State	946
14.9	Free Energy of Free Scalar Fields	947
14.10	Interacting Nonrelativistic Fields	949
14.10.1	Functional Formulation	951
14.10.2	Grand-Canonical Ensembles at Zero Temperature	952
14.11	Interacting Relativistic Fields	958
14.12	Plasma Oscillations	960
14.12.1	General Formalism	960

14.12.2	Physical Consequences	964
14.13	Pair Fields	968
14.14	Competition of Plasmon and Pair Fields	975
14.15	Ambiguity in the Selection of Important Channels	977
14.16	Gauge Fields and Gauge Fixing	981
14.17	Nontrivial Gauge and Faddeev-Popov Ghosts	989
14.18	Functional Formulation of Quantum Electrodynamics	992
14.18.1	Decay Rate of Dirac Vacuum in Electromagnetic Fields	993
14.18.2	Constant Electric and Magnetic Background Fields	997
14.18.3	Decay Rate in a Constant Electromagnetic Field	1000
14.18.4	Effective Action in a Purely Magnetic Field	1001
14.18.5	Effective Action in a Purely Magnetic Field	1001
14.18.6	Effective Action in a Purely Magnetic Field	1002
14.18.7	Heisenberg-Euler Lagrangian	1003
14.18.8	Alternative Derivation for a Constant Magnetic Field	1006
Appendix 14A	Propagator of the Bilocal Pair Field	1010
Appendix 14B	Fluctuations around the Composite Field	1012
Appendix 14C	Two-Loop Heisenberg-Euler Effective Action	1014
	Notes and References	1015
15	Systematic Graphical Construction of Feynman Diagrams	1019
15.1	Generalized Scalar ϕ^4 -Theory	1020
15.2	Basic Graphical Operations	1022
15.2.1	Cutting Lines	1022
15.2.2	Removing Lines	1025
15.3	Perturbation Theory for Vacuum Energy	1025
15.4	Functional Differential Equation for Free Energy in Scalar Theory	1027
15.5	Recursion Relation and Graphical Solution in Scalar Theory	1028
15.6	Scalar Yukawa-like Theory	1031
15.7	Perturbation Theory for the Yukawa-like Theory	1032
15.8	Functional Differential Equation for the Free Energy in Yukawa-like Theory	1032
15.9	Recursion Relation and Graphical Solution in the Yukawa-like Theory	1033
15.10	Matrix Representation of Diagrams	1035
15.11	Practical Generation	1037
15.11.1	Connected Vacuum Diagrams	1037
15.11.2	Two- and Four-Point Functions from Cutting Lines	1040
15.11.3	Two- and Four-Point Function from Removing Lines	1041
Appendix 15A	Tables	1043
	Notes and References	1052

16 Spontaneous Symmetry Breakdown	1053
16.1 Scalar $O(N)$ -Symmetric ϕ^4 -Theory	1053
16.2 Nambu-Goldstone Particles	1060
16.2.1 The Mechanism	1060
16.2.2 General Considerations	1061
16.2.3 Experimental Consequences	1063
16.3 Domain Walls in the $O(1)$ -Symmetric Theory	1064
16.4 Vortex Lines in the $O(2)$ -Symmetric Theory	1069
Notes and References	1076
17 Scalar Quantum Electrodynamics	1077
17.1 Action and Generating Functional	1077
17.2 Meissner-Ochsenfeld-Higgs Effect	1080
17.3 Spatially Varying Ground States	1087
17.4 Two Natural Length Scales	1088
17.5 Planar Domain Wall	1090
17.6 Surface Energy	1095
17.7 Single Vortex Line and Critical Field H_{c1}	1096
17.8 Critical Field H_{c2} where Superconductivity is Destroyed	1102
17.9 Order of Superconductive Phase Transition	1106
17.10 Quartic Interaction and Tricritical Point	1106
17.11 Four-Dimensional Version	1108
17.12 Spontaneous Mass Generation in a Massless Theory	1110
Notes and References	1111
18 Exactly Solvable $O(N)$-Symmetric ϕ^4-Theory for Large N	1112
18.1 Introduction of a Collective Field	1112
18.2 The Limit of Large N	1115
18.3 Variational Equations	1121
18.3.1 Non-trivial Ground States	1123
18.4 Special Features of Two Dimensions	1127
18.5 Experimental Consequences	1128
18.6 Correlation Functions for Large N	1132
18.7 No-Tachyon Theorem	1134
Notes and References	1134
19 Nonlinear σ-Model	1136
19.1 Definition of Classical Heisenberg Model	1136
19.2 Spherical Model	1139
19.3 Free Energy and Gap Equation in $D > 2$ Dimensions	1140
19.3.1 High-Temperature Phase	1142
19.3.2 Low-Temperature Phase	1143
19.4 Approaching the Critical Point	1145
19.5 Physical Properties of the Bare Temperature	1146

19.6	Spherical Model on Lattice	1148
19.7	Background Field Treatment of Cold Phase	1152
19.8	Quantum Statistics at Nonzero Temperature of Nonlinear σ -Model	1154
19.8.1	Two-Dimensional Model	1155
19.8.2	Four-Dimensional Model	1159
19.8.3	Temperature Behavior in Any Dimension	1160
19.9	Criteria for the Onset of Fluctuations in Ginzburg-Landau Theories	1165
19.9.1	Ginzburg's Criterion	1166
19.9.2	Azimuthal Correction to Ginzburg's Criterion	1167
19.9.3	Experimental Consequences	1169
	Notes and References	1170
20	The Renormalization Group	1172
20.1	Example for Redundancy in Parametrization of Renormalized Theory	1173
20.2	Renormalization Scheme	1175
20.3	The Renormalization Group Equation	1177
20.4	Calculation of Coefficient Functions from Counter Terms	1178
20.5	Solution of Renormalization Group Equations for Vertex Functions	1182
20.6	Renormalization Group for Effective Action and Effective Potential	1185
20.7	Approach to Scaling	1188
20.8	Explicit Solution of RGE Close to $D = 4$ Dimensions	1190
20.9	Further Critical Relations	1193
20.10	Comparison of Scaling Relations with Experiments	1197
20.11	Higher-Order Expansion	1199
20.12	Mean-Field Results for Critical Indices	1201
20.13	Effective Potential in the Critical Regime to Order ε	1203
20.14	$O(N)$ -Symmetric Theory	1208
20.15	Direct Scaling Form in the Limit of Large $N \rightarrow \infty$	1211
20.16	QED and Landau Ghosts	1212
	Notes and References	1214
21	Critical Properties of Nonlinear σ-Model	1215
21.1	Introductory Remarks	1215
21.2	Perturbation Theory	1217
21.3	Symmetry Properties of the Renormalized Effective Action	1222
21.4	Critical Behavior in $D = 2 + \varepsilon$ Dimensions	1225
21.5	Critical Exponents	1226
21.6	Two- and Three-Loop Results	1232
21.7	Variational Resummation of ε -Expansions	1235
21.7.1	Strong-Coupling Theory	1236
21.7.2	Interpolation	1239
21.8	Relation of σ -Model to Quantum Mechanics of a Point Particle on a Sphere	1245
21.9	Generalization of the Model	1248

Notes and References	1250
22 Functional-Integral Calculation of Effective Action. Loop Expansion	1253
22.1 General Formalism	1253
22.2 Quadratic Fluctuations	1256
22.3 Massless Theory and Widom Scaling	1265
22.4 Critical Coupling Strength	1267
22.5 Resumming the Effective Potential	1270
22.6 Fractional Gross-Pitaevskii Equation	1272
22.7 Summary	1273
Appendix 22A Effective Action to Second Order in \hbar	1274
Appendix 22B Effective Action to All Orders in \hbar	1277
Notes and References	1279
23 Exactly Solvable $O(N)$-Symmetric Four-Fermion Theory in $2 + \epsilon$ Dimensions	1281
23.1 Four-Fermion Self-Interaction	1281
23.2 Spontaneous Symmetry Breakdown	1286
23.3 Dimensionally Transmuted Coupling Constant	1287
23.4 Scattering Amplitude for Fermions	1289
23.5 Nonzero Bare Fermion Mass	1295
23.6 Pairing Model and Dynamically Generated Goldstone Bosons	1297
23.7 Spontaneously Broken Symmetry	1304
23.8 Relation between Pairing and Gross-Neveu Model	1307
23.9 Comparison with the $O(N)$ -Symmetric ϕ^4 -Theory	1309
23.10 Two Phase Transitions in the Chiral Gross-Neveu Model	1313
23.11 Finite-Temperature Properties	1316
Notes and References	1327
24 Internal Symmetries of Strong Interactions	1330
24.1 Classification of Elementary Particles	1330
24.2 Isospin in Nuclear Physics	1334
24.3 Isospin in Pion Physics	1338
24.4 $SU(3)$ -Symmetry	1341
24.5 Newer Quarks	1361
24.6 Tensor Representations and Young Tableaux	1362
24.7 Effective Interactions among Hadrons	1367
24.7.1 The Pion-Nucleon Interaction	1367
24.7.2 The Decay $\Delta(1232) \rightarrow N\pi$	1370
24.7.3 Vector Meson Decay $\rho(770) \rightarrow \pi\pi$	1373
24.7.4 Vector Meson Decays $\omega(783) \rightarrow \rho\pi$ and $\omega(783) \rightarrow \pi\pi\pi$	1374
24.7.5 Vector Meson Decays $K^*(892) \rightarrow K\pi$	1374
24.7.6 Axial Vector Meson Decay $a_1(1270) \rightarrow \rho\pi$	1375

24.7.7	Coupling of $\rho(770)$ -Meson to Nucleons	1376
Appendix 24A	Useful SU(3)-Formulas	1377
Appendix 24B	Decay Rate for $a_1 \rightarrow \rho\pi$	1379
Notes and References	1379
25	Symmetries Linking Internal and Spacetime Properties	1381
25.1	Approximate SU(4)-Symmetry of Nuclear Forces	1381
25.2	Approximate SU(6)-Symmetry in Strong Interactions	1388
25.3	From SU(6) to Current Algebra	1399
25.4	Supersymmetry	1405
Notes and References	1407
26	Hadronization of Quark Theories	1408
26.1	Introduction	1408
26.2	Abelian Quark Gluon Theory	1410
26.3	The Limit of Heavy Gluons	1427
26.4	More Quarks	1442
26.5	Summary	1443
26.6	Baryons	1443
Appendix 26A	Remarks on the Bethe-Salpeter Equation	1444
Appendix 26B	Vertices for Heavy Gluons	1448
Appendix 26C	Some Algebra	1450
Notes and References	1452
27	Weak Interactions	1457
27.1	Fermi Theory	1457
27.2	Lepton-Number Conservation	1461
27.3	Cabibbo Angle	1462
27.4	Cabibbo Mass Matrix	1463
27.5	Heavy Vector Bosons	1464
27.6	Standard Model of Electroweak Interactions	1465
27.7	Masses from Meissner-Ochsenfeld-Higgs Effect	1469
27.8	Lepton Masses	1471
27.9	More Leptons	1471
27.10	Weak Interaction of Hadrons	1472
27.11	Quantum Oscillations	1474
27.11.1	Oscillations between Neutral Kaons	1474
27.11.2	Mesons containing the Bottom Quark	1477
27.11.3	General Flavor Mixing	1477
27.12	Neutrino Mixing	1478
27.13	Simple Theory of Two-Neutrino Mixing	1479
27.14	Experiments	1480
27.15	Entangled Wavefunction	1482
Notes and References	1483

28 Nonabelian Gauge Theory of Strong Interactions	1486
28.1 Local Color Symmetry	1486
28.2 Gluon Action	1488
28.3 Quantization in the Coulomb Gauge	1489
28.4 General Functional Quantization of Gauge Fields	1496
28.5 Equivalence of Landau and Coulomb Gauges	1502
28.6 Perturbative QCD	1505
28.7 Approximate Chiral Symmetry	1507
Notes and References	1509
29 Cosmology with General Curvature-Dependent Lagrangian	1511
29.1 Simple Curvature-Saturated Model	1512
29.2 Field Equations of Curvature-Saturated Gravity	1514
29.3 Effective Gravitational Constant and Weak-Field Behavior	1516
29.4 Bicknell's Theorem	1516
Appendix 29A Newtonian Limit in a Nonflat Background	1518
Notes and References	1520
30 Einstein Gravity from Fluctuating Conformal Gravity	1522
30.1 Classical Conformal Gravity	1524
30.2 Quantization	1525
30.3 Outlook	1535
Appendix 30A Some Algebra	1535
Appendix 30B Quantization without Tachyons	1536
Notes and References	1542
31 Purely Geometric Part of Dark Matter	1545
Notes and References	1552
Index	1555

List of Figures

1.1	Probability distribution of a particle wave behind a double-slit . . .	12
1.2	Relevant function $\sum_{n=-N}^N e^{2\pi i \mu n}$ in Poisson's summation formula . .	30
1.3	Illustration of time-ordering procedure	37
1.4	Triangular closed contour for a Cauchy integral	78
2.1	Average Bose occupation number	123
2.2	Average Fermi occupation number	123
2.3	Temperature behavior of the specific heat of a free Fermi gas	131
2.4	Temperature behavior of the chemical potential of a free Bose gas .	132
2.5	Temperature behavior of the fraction of zero-momentum bosons in a free Bose gas	133
2.6	Temperature behavior of the specific heat of a free Bose gas	135
2.7	Rotating trap potential for $\omega^2 > 0$ and $\omega^2 < 0$	142
2.8	Contour C in the complex z -plane	148
2.9	Finite-size corrections to the critical temperature for $N > 300$. . .	172
3.1	Typical values of mass, s -wave scattering length $a \equiv a_s$, and number of atoms in a condensate of various atomic gases	178
3.2	Plot of the quasiparticle energies as function of momenta in an in- teracting Bose gas	187
3.3	Common volume of two spheres at a distance \mathbf{q} in momentum space	193
3.4	Energy density of an electron gas in uniform background of positive charge	195
3.5	Historical evolution of critical temperatures of superconductivity . .	197
3.6	Approximate energy of a free electron near the Fermi surface in a grand-canonical ensemble	199
3.7	Gap in the energy spectrum caused by attraction of pairs of electrons with opposite spin and momenta	200
3.8	Detail of the gap in energy spectrum	201
3.9	Solution of the gap equation for a weak attraction between electrons	210
3.10	Plot of the gap function and of the chemical potential	211
3.11	Temperature dependence of the normal fraction $\rho_{\mathbf{n}}/\rho$ in a Bose gas .	215
3.12	Reduced gap $s \equiv \bar{\Sigma}/\varepsilon_a$ as a function of the reduced s -wave scattering length $\hat{a}_s = 8\pi a_s/a = 8\pi a_s \rho^{1/3}$	223

3.13	Reduced energy per particle $w_1^e = W_1/N\varepsilon_a$ as a function of the reduced s -wave scattering length, compared with Bogoliubov's weak-coupling result	223
3.14	Temperature dependence of the normal particle density	231
3.15	Diagrams picturing the Wick contractions	233
4.1	Six leptons and quarks	312
4.2	Asymmetry observed in the distribution of electrons from the β -decay of polarized ${}^{60}_{27}\text{Co}$	314
4.3	Effect of raising and lowering operators \hat{L}_+ and \hat{L}_- upon the states $ s, m\rangle$	357
4.4	Triangle formed by rapidities in a hyperbolic space. The sum of angles is smaller than 180° . The angular defect yields the angle of the Thomas precession	378
5.1	Two equal masses M oscillating at the ends of a spring as a source of gravitational radiation	407
5.2	Two spherical masses in circular orbits around their center of mass	409
5.3	Gravitational amplitudes arriving on Earth from possible sources	411
5.4	Shift of time of the periastron passage of PSR 1913+16	412
5.5	Two pulsars orbiting around each other	413
5.6	Two masses in a Keplerian orbit around the common center-of-mass	413
5.7	Energy emitted by two point-masses on a circular orbit around each other	414
5.8	Particle falling radially towards a large mass	415
5.9	Spectrum of the gravitational radiation emitted by a particle of mass m falling radially into a black hole of mass M	416
5.10	Distortions of a circular array of mass points caused by the passage of a gravitational quadrupole wave	421
5.11	Field lines of tidal forces of a gravitational wave	422
5.12	Two chirps detected by the LIGO collaboration	424
5.13	Gyroscope carrying a frame x', z' around a polar orbit with a fixed orientation with respect to the fixed stars	432
6.1	Hydrogen spectrum according to Dirac's theory	458
7.1	Pole positions in the complex p^0 -plane in the integral representations of Feynman propagators	495
7.2	Wick rotation of the contour of integration in the complex p^0 -plane	495
7.3	Integration contours in the complex p^0 -plane of the Fourier integral for various propagators	507
7.4	Different coupling schemes for two-particle states of total angular momentum j and helicity m	519
7.5	Geometry of the silver plates for the calculation of the Casimir effect	579

9.1	Behavior of wave function for different positions of a bound state near the continuum	695
9.2	Behavior of binding energy and scattering length in an attractive square-well potential	699
9.3	Geometry of particle beams in a collider	706
11.1	Singularities in the complex q_0 -plane of a Feynman propagator . . .	797
12.1	An electron on the mass shell absorbing several photons	816
12.2	An electron on the mass shell absorbing several photons, plus one additional photon	816
12.3	An internal electron loop absorbing several photons, plus an additional photon, and leaving again on the mass shell	817
12.4	Transition of an atomic state from a state n with energy E_n to a lower state n' with energy $E_{n'}$, thereby emitting a photon with a frequency $\omega = (E_{n'} - E_n)/\hbar$	823
12.5	Kinematics of Rutherford scattering	828
12.6	Lowest-order Feynman diagrams contributing to Compton Scattering and giving rise to the Klein-Nishina formula	833
12.7	Illustration of the photon polarization sum in Compton scattering .	837
12.8	Ratio between total relativistic Compton cross section and nonrelativistic Thomson cross section	840
12.9	Lowest-order Feynman diagrams contributing to electron-positron annihilation	840
12.10	Illustration of the photon polarization sum in electron-positron annihilation	843
12.11	Electron-positron annihilation cross section	844
12.12	Lowest-order Feynman diagrams contributing to the decay of parapositronium decay	845
12.13	Lowest-order Feynman Diagrams contributing to decay of orthopositronium decay	846
12.14	Trajectories in the simplest classical Bremsstrahlung process: An electron changing abruptly its momentum	847
12.15	Lowest-order Feynman diagrams contributing to Bremsstrahlung. The vertical photon line indicates the nuclear Coulomb potential . .	850
12.16	The angles θ' , θ , φ in the Bethe-Heitler cross section formula	851
12.17	Lowest-order Feynman diagrams contributing to electron-electron scattering	852
12.18	Kinematics of electron-electron scattering in the center of mass frame	853
12.19	General form of diagrams contributing to electron-positron scattering	854
12.20	Lowest-order contributions to electron-positron scattering	855
12.21	Experimental data for electron-electron and electron-positron scattering at $\theta = 90^\circ$ as a function of the incident electron energy	856
12.22	Cross section for Bhabha scattering at high energy	857

12.23	Vertex correction responsible for the anomalous magnetic moment	858
12.24	Leading hadronic vacuum polarization corrections to a_μ	866
12.25	One-loop electroweak radiative corrections to a_μ	867
12.26	Measured values of a_μ and prediction of the Standard Model (SM)	868
12.27	Lowest-order Feynman diagram for the vacuum polarization	868
12.28	Lowest-order Feynman diagram for the self-energy of the electron	875
12.29	Diagrammatic content in the calculation of the energy shift via Schrödinger wave function	886
13.1	Graphical solution of the recursion relation (13.30) for the generating functional of all connected Green functions	906
13.2	Tree decomposition of connected Green functions into one-particle irreducible parts	911
13.3	Graphical solution of the functional differential equation (13.64)	914
13.4	Recursion relation for two-particle-irreducible graphs in the effective action	920
13.5	The anharmonic model integral Z as a function of $g' = g/\omega^4$	923
13.6	Approximations to βF obtained from the extrema of the higher ef- fective action	924
14.1	Pure-current term of the collective action	963
14.2	Non-polynomial self-interaction terms of plasmons	963
14.3	Free plasmon propagator	964
14.4	Fundamental particles entering any diagram only via the external currents	970
14.5	Free pair field following the Bethe-Salpeter equation	972
14.6	Free pair propagator	974
14.7	Self-interaction terms of the non-polynomial pair action	975
16.1	Effective potential of the ϕ^4 -theory for $N = 2$ in mean-field approx- imation	1056
16.2	Magnetization Φ_0 in mean-field approximation as a function of the temperature ratio T/T_c^{MF}	1057
16.3	Magnetization Φ_j as a function of the external source j in mean-field approximation	1059
16.4	Plot of the symmetric double-well potential	1065
16.5	Classical kink solution in double-well potential connecting the two degenerate maxima in the reversed potential	1066
16.6	Reversed double-well potential governing the motion of the position ϕ as a function of the imaginary time x	1067
16.7	Reduced order parameter $\bar{\rho} = \varphi / \varphi_0 $ around a vortex line	1071
17.1	Dependence of order parameter ρ and magnetic field H on the re- duced distance z between the normal and superconductive phases	1092
17.2	Order parameter ρ and magnetic field h for a vortex line	1100

17.3	Critical field h_{c1} where a vortex line of strength n begins invading a type-II superconductor	1101
17.4	Spatial distribution magnetization of the order parameter $\rho(\mathbf{x})$ in a typical mixed state in which the vortex lines form a hexagonal lattice	1102
17.5	Effective potential for the order parameter ρ with fluctuation-generated cubic term	1107
17.6	Effective potential for the order parameter ρ in four spacetime dimensions	1109
19.1	Free energy as a function of λ for $D = 2$	1141
19.2	Free energy as a function of λ for $D > 2$	1143
19.3	Solution of the gap equation (19.48) for $\epsilon = 1$ and large volume L^D	1145
19.4	Temperature behavior of the correlation length	1146
20.1	Curves in the (μ, g) -plane corresponding to the same physical fermion mass	1175
20.2	Flow of the coupling constant $g(\sigma)$ as the scale parameter σ approaches zero (infrared limit)	1190
20.3	Flow of the coupling constant α_μ as the scale parameter μ increases (ultraviolet limit)	1213
21.1	Two-loop diagrams	1232
21.2	Three-loop diagrams	1232
21.3	Integrands of the Padé-Borel transform for the Padé approximants .	1236
21.4	Inverse of the critical exponent ν for the classical Heisenberg model in the O(3)-universality class plotted as a function of $\epsilon = 4 - D$. . .	1243
21.5	Inverse of the critical exponent ν for the O(3)-universality class plotted as a function of $\epsilon = 4 - D$	1244
21.6	Inverse of the critical exponent ν for the O(5)-universality class plotted as a function of $\epsilon = 4 - D$	1244
21.7	Highest approximations ($M = 4$) for $n = 3, 4, 5$, and the $1/n$ -expansions to order $1/n^2$	1244
21.8	Inverse of the critical exponent ν for the O(1)-universality class (of the Ising model) plotted as a function of $\epsilon = 4 - D$	1245
22.1	Solution of the variational equation (22.118)	1272
22.2	Condensate density from the Gross-Pitaevskii equation and its fractional version	1274
23.1	One-loop Feynman diagram in the inverse propagator of the σ' -field	1289
23.2	Function $J(z) + 2$ in the denominator of the σ' -propagator	1291
23.3	Two transition lines in the N - g -plane of the chiral Gross-Neveu model in $2 + \epsilon$ dimensions	1316
23.4	Solution of the temperature-dependent gap equation	1320

24.1	Total and elastic π^+ -proton cross section	1332
24.2	Total and elastic π^- -proton cross section	1333
24.3	Photon-proton and photon-deuteron total cross sections	1334
24.4	Mirror nuclei ${}_5\text{B}^{11}$ and ${}_6\text{C}^{11}$ with their excited states	1335
24.5	Singlets and triplets of isospin in the nuclei ${}_6\text{C}^{14}$, ${}_7\text{N}^{14}$, ${}_8\text{O}^{14}$	1337
24.6	Pseudoscalar meson octet states associated with the triplet of pions. The same picture holds for the vector meson octet states with the replacement (24.62)	1343
24.7	Baryon octet states associated with nucleons	1344
24.8	Baryon decuplet states associated with the first resonance of nucleons	1345
24.9	Quark content of the pseudoscalar meson octet	1347
24.10	Effect of raising and lowering operators on quark and antiquark states	1350
24.11	Addition of the fundamental weights in product representation space of 3 and $\bar{3}$ vectors	1351
24.12	States of the $\bar{3}$ -representation	1352
24.13	Quark-antiquark content of the meson octet	1353
24.14	Combination of indices a in the pseudoscalar octet field M_a^\dagger	1354
24.15	Quark content in the reduction of the product $3 \times 3 = 6 + \bar{3}$	1355
24.16	Octet and singlet states obtained from $3 \times \bar{3}$	1356
24.17	Irreducible three-quark states 10 and 8 in the product 3×6	1357
24.18	The four quarks u, d, s, c and their position in the three-dimensional weight space	1362
25.1	Would-be SU(4) -partner of the deuteron, with spin-1 and isospin-0 .	1385
25.2	Pseudoscalar and vector mesons of the 35-representation of SU(6) .	1390
25.3	SU(3)-content of particles in the 56-representation of SU(6)	1391
25.4	Nucleon resonances of negative parity in the 70-representation of SU(6)	1392
25.5	Octet of spin-parity $\frac{1}{2}^+$ -baryons	1393
26.1	Ladder diagrams summed by a Bethe-Salpeter equation	1417
26.2	Ladder diagrams summed in the tadpole term	1418
26.3	Rainbow diagrams in the tadpole term	1418
26.4	Ladder of gluon exchanges summed in a meson tadpole diagram . .	1419
26.5	Gluon diagrams contained in a three-meson vertex	1421
26.6	Three-meson vertex drawn in two alternative ways	1422
26.7	Quark-gluon exchanges summed in meson exchange diagrams	1423
26.8	Quark-gluon diagrams summed in a meson loop diagram	1424
26.9	Multi-meson emission from a quark line	1424
26.10	Twisted exchange of a meson between two quark lines	1425
26.11	Vector meson dominance in the coupling of an external photon to a quark line	1425
26.12	Vector meson dominance in a photon propagator	1426
26.13	Gluon diagrams in a meson propagator	1426

26.14	Diagrams in the Bethe-Salpeter equation	1445
26.15	Momenta in the integral equation	1445
27.1	Quark diagrams for K^+ and K^0 decays involving strangeness changing neutral currents	1473
27.2	Diagrams for the $K^0 \rightarrow \mu^+ \mu^-$ decay with compensating strangeness-changing neutral currents	1473
27.3	Oscillation of decay rate into $\pi^+ \pi^-$ of K^0 -beam	1476
27.4	Asymmetry of the number of mesons as a function of time	1476
27.5	Oscillations of decay rate for the processes (148127.141148027.5) and (148127.142148027.5). The period is in both cases roughly 7 sec. The inserts show the frequency analyses. Plots are from Ref. [27].	1481
27.6	The upper KamLAND regime of 2006 [36] is compatible with the present result $\Delta m^2 \approx 22.5 \times 10^{-5} \text{eV}^2$	1481
28.1	Propagators in the Yang-Mills theory	1502
28.2	Vertices in the Yang-Mills theory	1502
28.3	Flow of the coupling constant α_s towards the origin as the scale parameter μ approaches infinity (ultraviolet limit)	1507
29.1	Curvature-saturated Lagrangian	1514
29.2	Effective gravitational constant	1516
29.3	Potential $V(\psi)$ associated with the curvature-saturated action via Bicknell's theorem	1518
30.1	Calculation of Feynman propagator	1537
30.2	Calculation of Feynman propagator without tachyons	1539
31.1	Details of the fits to the velocity data. Filled triangles refer to the northern half of the galaxy, open squares to the southern half.	1545
31.2	Velocity curve (points) of the galaxy M33 and comparison with a best fit model calculation	1546
31.3	Various types of matter in the universe	1546
31.4	Various contributions to Dark Matter	1547

List of Tables

4.1	Transformation properties of various composite fields	325
4.2	Lowest Clebsch-Gordan coefficients	381
5.1	Binary systems as sources of gravitational radiation	410
5.2	Some observed parameters of PSR 1913+16	412
5.3	Typical astrophysical sources of gravitational radiation	417
12.1	Different contributions to $a_{\mu}^{\text{str}}(\text{vac. pol.})$ in the integral 12.433	867
15.1	Vacuum diagrams. Connected diagrams with their multiplicities in the ϕ^4 -theory with their multiplicities up to five loops	1043
15.2	Two-point functions. Connected diagrams with their multiplicities in the ϕ^4 -theory up to four loops	1044
15.3	Four-point functions. Connected diagrams with their multiplicities in the ϕ^4 -theory up to three loops	1045
15.4	Vacuum diagrams. Connected graphs with their multiplicities in the Yukawa-like $\phi^2 A$ -theory	1047
15.5	Connected <i>vacuum</i> diagrams. Unique matrix representation	1048
15.6	Connected <i>two-point</i> functions. Unique matrix representation	1049
15.7	Connected <i>four-point</i> functions. Unique matrix representation	1050
17.1	Different critical magnetic fields for various superconducting materials	1105
19.1	Values of the lattice Yukawa potential $v_l^D(\mathbf{0})$ of mass l^2 at the origin for different dimensions and l^2	1149
21.1	Coefficients $b_n(\hat{g}_0)$ of the strong-coupling expansion	1239
21.2	Coefficients of the successive extension of the expansion coefficients for $n = 3$	1241
21.3	Coefficients of the successive extension of the expansion coefficients for $n = 4$	1241
21.4	Coefficients of the successive extension of the expansion coefficients for $n = 5$	1241
21.5	Coefficients of the successive extension of the expansion coefficients for $n = 1$	1241
24.1	Masses and lifetimes of the octet states associated with the isodoublet of nucleons	1344

24.2	Structure constants of $SU(3)$	1348
24.3	The symmetric couplings d_{abc}	1349
24.4	List of Quarks and their properties.	1361
24.5	Isoscalar factors of $SU(3)$	1378
25.1	Action of the different interchange operators	1382
25.2	Action of spin and isospin operators in the expansion (25.8)	1383
25.3	Eigenvalues of charge and other operators on quark states	1394
27.1	List of leptons and their properties.	1472

Visualization of a quantum field theory calculation showing virtual particles in the quantum vacuum. Even in empty space, this vacuum energy is non-zero, but without specific boundary conditions, individual particle properties will not be constrained. (DEREK LEINWEBER). So if everything is fields, then what is a particle? You may have heard a phrase before: that particles are excitations of quantum fields. In other words, these are quantum fields not in their lowest-energy "or zero-point" state, but in some higher-energy state. But exactly how this works is a bit tricky. Up until this point,