PARTICLES AND QUANTUM FIELDS

Particles and Quantum Fields

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To my wife Annemarie
and our son Hagen Michael II

Preface

This book arose from lectures I gave at the Freie Universität Berlin over the past five decades. They were intended to prepare graduate students for their research in elementary-particle physics or in many-body theory of condensed matter. They should serve as a general introduction and a basis for understanding more advanced work on the subject.

The theory of quantum fields presented in this book is mainly based on the perturbative approach. Elementary particles are introduced initially without any interactions. These are added later, and their strength is parametrized by some coupling constant g. The consequences are studied order by order in g, with the particles propagating forward from interaction to interaction. Such a treatment is clearly a gross simplification of what happens in nature, where even the existence of a free particle involves the full interaction from the very beginning. Nevertheless, this kind of procedure has been the basis of many successful theories. In all of them, there exist dominant freely propagating excitations or elementary particles at least in some experimentally accessible limit. The most prominent example is the theory of strongly interacting particles. There they are described as being composed of quarks held together by gluons which interact via a nonabelian gauge theory called quantum chromodynamics (QCD). In the limit of large energies, the particles behave like free point-like particles. This behavior was named *parton-like* by Richard Feynman. The existence of such a limiting behavior in QCD was called *asymptotic freedom*. It was the main reason for the possibility of developing a theory for these particles, which gave good explanations of many interaction processes between elementary particles. The initial *creation* of the particles, however, remained far from being understood. It involves a regime of strong interactions where perturbation theory fails.

A field-theoretic method to reach into this regime has been developed in quantum field theory of many-body physics. There a separation of the two regimes, the formation of particle-like excitation and their scattering, is much clearer to observe experimentally than in particle physics. For this reason, many-body theory has been a major source of inspiration for the development of theoretical methods to describe strongly interacting phenomena. An extension of perturbation theory into the strongly interacting regime has so far been possible mainly by employing resummation techniques. Initially, theorists have summed certain classes of Feynman diagrams by repeating infinitely many times the same interaction leading to a geometric series whose sum can be evaluated. This has allowed the understanding of many pronounced observable phenomena as consequences of a sum of infinitely many bubbles and ladders of diagrams. The methods for this were developed by Hartree, Fock, and Bogoliubov in many-body theory, and by Bethe and Salpeter in quantum electrodynamics.

The development of renormalization group theory has led to a generalization of this method. It permits to extend the sum of bubbles and ladders to sums of diagrams of many different topologies. This makes them applicable in the regime of strong couplings, where they can be used to study various many-body phenomena even in the so-called *critical regime*. There the interactions become so strong that they are much more important than the free-particle propagation.

In many-body theory, one can parametrize the separation of the two regimes quite clearly by formulating the theory on a lattice. The propagation is characterized by a so-called hopping amplitude from lattice point to lattice point. The critical regime is reached when the masses of some of the participating excitations go to zero. In this limit, the range of their propagation tends to infinity, and their interaction becomes increasingly important.

An efficient alternative to the summation of infinitely many perturbation-theoretic diagrams is based on a variational approach. Its power was discovered in 1877 by John Rayleigh and formalized by Walter Ritz in 1908. Some time ago, the theory was revived by Feynman and Kleinert.[1] They set up a first-order variational approximation to path integrals, which led to reasonable approximations for a variety of quantum mechanical problems. The approximations were later expanded to all orders, and have finally led to the the powerful *field-theoretic variational perturbation theory* (VPT). In that form, the theory is able to simplify and replace the popular renormalization group approach of critical phenomena. It has been successfully applied to many phase transitions, and is published in a monograph.[2]

An important aspect of a theory of critical phenomena is the fact that the free-field propagators play no longer the important role they have in perturbation expansions. The underlying free-particle behavior is based on a Gaussian approximation to field fluctuations. In the critical regime, this approximation of the distributions has tails which follow power-like distributions. Such tails are observed in the statistics of very rare events, which are called “black-swan events”.[3] These occur in nature in many different circumstances, ranging from oceanic monster waves over earthquakes and wind gusts, to catastrophic crashes of financial markets.[4]

I want to thank my friend Remo Ruffini for creating an extremely lively and inspiring environment for scientific work in particle and astrophysics at many exciting places of the globe, where I was invited for lectures and discussions of topics of this

[1]R.P. Feynman and H. Kleinert, Phys. Rev. A *34*, 5080 (1986).

[2]H. Kleinert and V. Schulte-Frohlinde, *Critical Properties of Φ^4-Theories*, World Scientific, Singapore 2001, pp. 1–489 (`http://klnrt.de/b8`). See Chapter 20 for the variational approach.

[3]H. Kleinert, *Quantum Field Theory of Black-Swan Events*, EPL *100*, 10001 (2013) (`www.ejtp.com/articles/ejtpv11i31p1.pdf`); *Effective Action and Field Equation for BEC from Weak to Strong Couplings*, J. Phys. B *46*, 175401 (2013) (`http://klnrt.de/403`).

[4]H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, World Scientific, Singapore, 2009 (`http://klnrt.de/b5`). See Chapter 20.

book. Ruffini, who holds a chair in theoretical physics at the university of Rome "La Sapienza", founded an international center which I am part of, where scientists from all over the world do research, and where students can prepare their Ph.D. degree (for details see ICRANet.org).

I am also very grateful to my colleague Axel Pelster who, for many years, has shared with me the burden and joy of bringing students of the Freie Universität Berlin to their master's and doctor's degrees. His careful reading of large parts of the manuscript has produced useful insights and corrections.

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The reader who detects errors, is kindly asked to report them by email to h@klnrt.de.

Hagen Kleinert
Berlin, April 2016

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Visualization of a quantum field theory calculation showing virtual particles in the quantum vacuum. Even in empty space, this vacuum energy is non-zero, but without specific boundary conditions, individual particle properties will not be constrained. (DEREK LEINWEBER). So if everything is fields, then what is a particle? You may have heard a phrase before: that particles are excitations of quantum fields. In other words, these are quantum fields not in their lowest-energy â€” or zero-point â€” state, but in some higher-energy state. But exactly how this works is a bit tricky. Up until this point,