Computer Modeling of Coax Cable Circuits

Why fool around with approximations and guesses? You can know exactly what's happening on your coaxial cable.

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This article will try to bring under one roof the information that is needed to understand the behavior of, and calculate the performance of, ordinary types of coax cable that we commonly use in amateur-radio work. It's been my experience that this information resides in bits and pieces in numerous articles, handbooks and textbooks, but is somewhat tedious and confusing to pull together into a clear and simple overview that's easy to use to understand and design RF systems. The approach is to put you in direct contact with mathematical formulas that can be easily evaluated on a personal computer, for example in a *Mathcad* worksheet. [1] The results of the calculations can then often be used by the *ARRL Radio Designer* program and other software to perform simulations and optimizations of various kinds. I'll try to keep the mathematics fairly, but not perfectly, painless. For many, much of this material will be a review.

In this article meters and centimeters will be used, to be consistent with the prevailing literature. To convert feet and inches to meters and centimeters:

m	= f	x 0.30480	f^{-1}	$= m^{-1}$	x 0.30480
f	= m	x 3.2808	m^{-1}	$= f^{-1}$	x 3.2808
cm	= in	x 2.540	in^{-1}	$= cm^{-1}$	x 2.540
in	= cm	x 0.3937	cm^{-1}	$= in^{-1}$	x 0.3937

The Basic Idea of Coax

A length of coax conveys RF power. A step voltage applied to one end creates a power wave that reaches the load at some later time. Some of this power may return back to the sending end at a still later time. The two conductors (the center lead and the braid) are said to act as *guides* for this power wave, and that the electromagnetic energy is actually conveyed, in a mathematical sense, by the electric and magnetic fields that exist within the dielectric material. [2] After the initial, or *transient* conditions, have all been resolved, this traveling wave viewpoint is no longer mandatory and we find that in the *steady state* a length *l* of coax is a device that *transforms* impedances, voltages and currents in a way that is similar to, but not exactly like, a conventional transformer. We will focus on these steady-state conditions.

The key to understanding coax behavior lies in understanding the electric and magnetic fields. For example, the current flow in the center lead creates a magnetic field that encloses the current. The return current in the braid also creates a magnetic field that lies *entirely outside* the region of its current flow. These two flux fields cancel each other on the *outside* of the coax. The electric field exists only *between* the conductors. These are the principal shielding mechanisms that keep the signal inside the coax. Various imperfections detract from this goal, and the coax braid is not a perfect shield. An excellent reference on coax leakage is listed in Note 3. The braid also protects, to a large extent, the inside of the coax from external electric and magnetic fields. The exact electromagnetic theory principles by which these and many other effects occur are complicated and are beyond the scope of this article.

At radio frequencies, a segment of coax can function as a *conventional* transformer and is often used this way. The center lead can be a primary and the braid a secondary (and vice versa). The turns ratio in this mode is 1:1.

Characteristic Impedance

Fig 1 shows a length of coax connected to an impedance Z_0^* (the reason for this will become apparent later) that equals the complex conjugate of Z_0, the *dynamic characteristic impedance* of the cable. This means that as a power wave travels toward the load, the ratio of the voltage V^+ to the current I^+ is Z_0. At *any* frequency this ratio is:

$$Z_0 = \frac{V^+}{I^+} = \sqrt{\frac{R + j\,\omega\,L}{G + j\,\omega\,C}} \quad \Omega \qquad \text{Eq 1}$$

where *R* is the series resistance (Ω) per meter and *L* is the series inductance (H) per meter. *G* is the shunt conductance (S) per meter and *C* is the shunt capacitance (F) per meter of the polyethylene dielectric material. *R* and *G* are responsible for the power losses (attenuation) in the cable and also for a reactive component to Z_0.

This formula is meaningful to us only at radio frequencies where:

$$\omega L >> R \text{ and } \omega C >> G \qquad \text{Eq 2}$$

and as frequency $\omega(=2pf)$ increases, Z_0 would seem to be getting closer and closer to the simple approximate pure resistance value, which is nearly always used in most ordinary low-loss applications:

$$Z_0 \approx \sqrt{\frac{L}{C}} = R_0 \qquad \text{Eq 3}$$

but it turns out that *R* increases as the square root of frequency and *G* increases directly with frequency (more about this later), so that this ideal value is somewhat elusive.

In some situations, when the attenuation is not negligible but also not large, we would like to have a more accurate value of Z_0:

$$Z_0 \approx R_0 \left[1 - j\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)\right] \qquad \text{Eq 4}$$

and we see more clearly how the cable losses *R* and *G* cause Z_0 to be slightly reactive, which is a rather peculiar result. At mid frequencies the term $G/2\omega C$ is usually much less than $R/2\omega L$ (dielectric losses are small) and Eq 4 further simplifies to:

$$Z_0 \approx R_0 \left(1 - j\frac{R}{2\omega L}\right) \qquad \text{Eq 5}$$

but at the highest UHF frequencies, because *G* increases more rapidly than *R*, greater accuracy results if we use Eq 4. At some microwave frequency, coax (especially thick coax) starts to behave somewhat like a complicated waveguide, but we will not be concerned with that.

Later, we will revisit and improve these formulas as we develop additional information (see Eq 20). First, we will give some details regarding Z_0, *R*, *L*, *G* and *C* that are interesting and useful.

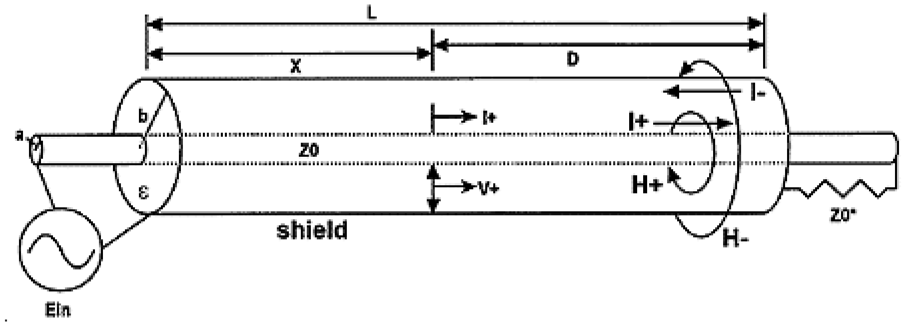


Fig 1—A coaxial cable of characteristic impedance Z_0 and length *L*, terminated in Z_0*. The currents and H-fields are shown.

Coax Cable Parameters

The inductance *L* per meter is determined by finding the magnetic flux ϕ per meter of length, between the center lead and the braid, that is generated by a current *I* in the center lead, according to Ampere's law. (Note: the return current through the braid does not generate flux in the dielectric region between the center wire and the braid.) The ratio of flux to current is, by definition, the inductance per meter (*N*=1, one single turn, is assumed):

$$L = 0.2 \ln\left(\frac{b}{a}\right) = 0.00333\ R_0\sqrt{\varepsilon_r} = 0.00333\frac{R_0}{VF}\ \ \mu\text{H/m} \qquad \text{Eq 6}$$

where ln is the natural logarithm. In this equation, *b* and *a* are shown in **Fig 1**, ε_r is the relative dielectric constant and *VF*, the velocity factor, is the ratio of the wave's phase velocity to the speed of light (in air or vacuum). This number, for many types of 50-Ω coax, is $VF \approx 0.64$ to 0.67, with 0.66 as typical. It follows from Eq 6 that:

$$\sqrt{\varepsilon_r} = \frac{1}{VF} \qquad \text{Eq 7}$$

which makes $\varepsilon_r \approx (1/0.66)^2$ or about 2.3 for the commonly used polyethylene dielectric material. For a 50-Ω coax, Eq 6 finds the *b/a* ratio to be:

$$\ln\left(\frac{b}{a}\right) = \frac{(0.00333)(50)}{(0.2)(0.66)} \quad \text{or} \quad \frac{b}{a} = e^{1.25} = 3.49 \qquad \text{Eq 8}$$

From Eq 6, a typical value for *L* is $0.00333 \times 50 \div 0.66 = 0.252$ µH per meter.

The capacitance *C* per meter is found by relating a charge in coulombs per meter on the center wire to the voltage difference between center lead and braid, according to Gauss's law. The result for coax is:

$$C = \frac{\varepsilon_r}{18 \ln\left(\frac{b}{a}\right)} \bullet 10^3 = \frac{\sqrt{\varepsilon_r}}{3R_0} \bullet 10^4 \quad \text{pF/m} \qquad \text{Eq 9}$$

A typical value of *C* for 50-Ω coax is approximately 101 pF per meter. Using these typical *L* and *C* values, the square root of 0.2525 µH / 101.1 pF gives a Z_0 of 50.0 Ω. Combining Eqs 6 and 9, we can now rewrite Eq 3 as follows:

$$Z_0 \approx \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right) = \frac{138}{\sqrt{\varepsilon_r}} \log_{10}\left(\frac{b}{a}\right) = R_0 \quad W \qquad \text{Eq 10}$$

The dielectric material of the cable has a capacitive susceptance $j\omega C$ per meter and is in parallel with a conductance *G* per meter. In **Fig 2** the very small angle *q* between *Y* and ωC is the *loss angle* and the tangent of that angle, $G/\omega C$, is the *loss tangent* T_L. A small angle implies a low value of *G* and for this small angle the loss tangent is practically the same as the *power factor* F_P, which is which is also used. These are specified for various dielectric materials (for polyethylene $\approx 5 \times 10^{-4}$) and so:

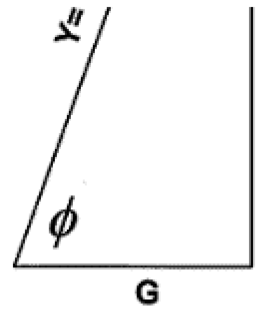
$$G = T_L \bullet \omega C \approx F_P \bullet \omega C \qquad \text{Eq 11}$$

As an example, for 1 meter of 50-Ω cable at 100 MHz $G \approx 3 \times 10^{-5}$ S. The dielectric losses are due mostly to hysteresis effects in the dielectric molecules as the electric field oscillates.

The resistance per meter, *R*, involves the center lead and the braid, with the resistance of the center lead being typically four times greater. [4] The basic resistivity of copper is 1.75×10^{-8} Ω-meter, which means that a cube of copper, one meter to a side, would have that value of resistance between two opposite faces of the cube. The formula for the resistance of any cube is:

$$R_{cube} = \frac{\rho}{l} \qquad \text{Eq 12}$$

where *r* is resistivity and *l* is the length of one side. A cube of 1 cm per side has 100 times as much resistance as a 1-meter cube.



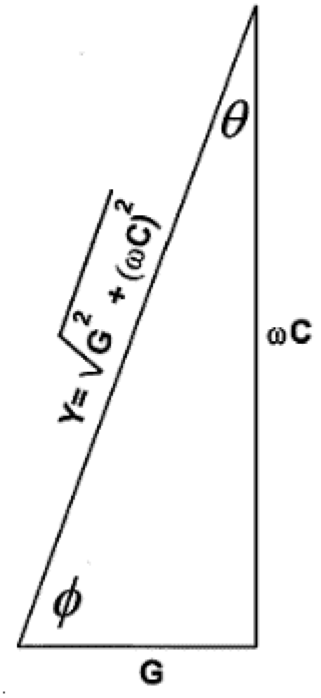


Fig 2—A graphical definition of loss tangent and power factor.

As frequency increases the resistance increases due to skin effect. That is, the current in the center conductor flows mostly along the outer surface. If the wire is tin-plated, a significant part of the current flows in the tin, whose resistivity is 6.7 times greater than copper. So low-loss coax avoids tin plating, despite its presumably easier soldering property (RG-11 is an example). If the center lead is stranded the resistance is about 1.3 times higher because of the spiraling of the strands and the contact resistance between adjacent strands, so a solid wire is preferred, but stranded is often used anyway because it is more flexible. Increasing the size of the center lead reduces its loss significantly but also reduces Z_0, according to Eq 10. To maintain the same Z_0 the cable diameter must be increased. If, instead, a foam dielectric with a lower ε_r =1.56 is combined with a larger conductor, Z_0 and the cable diameter can be maintained at the same size, but with lower copper loss. RG-8 foam (*VF*=0.80) is an example.

The braided shield also contributes to resistance. The current flows on the inner surface, also because of skin effect. The manner in which the wire is braided is important, and the outer jacket presses tightly against the braid to reduce resistance and improve stability under flexure. Manufacturers use *braid factors* to characterize their products.

Skin effect is a consequence of Ampere's law and Faraday's law. **Fig 3** illustrates the action. The rapidly changing magnetic flux within the loop a-b-c-d-a, due to current *inside* the wire (Ampere's law), induces an electric field (Faraday's law) within this loop. This electric field is in such a direction that the current in path a-to-b is retarded and the current in path d-to-c is enhanced. The induced electric field strength increases directly with frequency, forcing the current ever closer to the surface at path d-to-c. The result is that the interior of the wire is very nearly an insulator for the current in **Fig 3**. **Fig 3** also shows E-field components from a-to-d and from c-to-b, but these are canceled out by opposing E-fields in adjacent segments to the left and right. The same action occurs on the braid, forcing the current toward the inside surface of the braid.

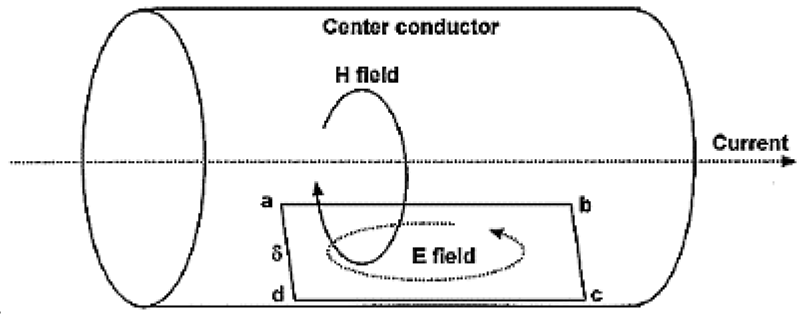


Fig 3—The actions of the E- and H-fields are responsible for skin effect.

At a distance *d* (the skin depth) from the surface the current density has reduced to the factor e^{-1} = 0.37 (37%) and at 5*d* it is e^{-5} = 0.0067 (0.67%), where e = 2.7183. The value of *d* for copper is:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu}} = \frac{6.64}{\sqrt{f}} \quad \text{cm}$$

Eq 13

where *m* is the magnetic permeability of copper ($4\pi \times 10^{-7}$ H/m), the same as free space. At 1.8 MHz, δ=0.049 mm or 0.0019 in.

Because of the magnetically induced electric field *inside* the wire, there is a phase lag in the *internal* current in the amount of 1.0 radian at a depth of *d*. This is called the *internal inductance* effect. This inductance is *not* the same as the *L* in Eq 1, but at low frequencies it makes a small contribution to it and it almost disappears at high frequency. For copper coax, the total resistance of the center wire plus braid is approximately (the vendor's correction factors are needed):

$$R = \frac{1}{2}\sqrt{\frac{\rho f \mu}{\pi}}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{4.17 \bullet 10^{-6}\sqrt{f}}{b}\left(\frac{b}{a}+1\right) \quad \Omega / \text{m}$$

Eq 14

where *a* and *b* are in *centimeters* (see **Fig 1**) and the resistance increases as the square root of *r* and *f*. The *internal* inductance *decreases* at the same rate. The ratio *b*/*a* determines Z_0 (see Eq 10) so for a given Z_0 a greater coax diameter *b* reduces *R*.

Wave Propagation (No Reflections)

Let's look at the behavior of the wave as it travels from the generator to the load. For starters, assume that the load impedance $Z_L = Z_0^*$ and both are almost resistive. In other words (see Eqs 4 and 5), the attenuation of the cable is very small. We are looking for the *matched* performance and there are negligible reflections from the load, as mentioned earlier.

If an ac voltage with rms value E_S is applied (**Fig 1**) the voltage *E* at point *x* is:

$$E(x) = \underbrace{E_S e^{-\gamma x}}_{A} = \underbrace{E_S e^{-\sqrt{(R+j\omega L)(G+j\omega C)} \cdot x}}_{B} = E_S \underbrace{e^{-\alpha x}}_{C} \underbrace{e^{-j\beta x}}_{D} ;$$

$$I(x) = \frac{E(x)}{Z_0}$$

Eq 15

This is the steady-state solution to a differential wave equation for a sine-wave signal that is traveling in *one* direction—toward the load. Term A contains γ, the *propagation constant* per meter. Term B shows that γ^2 is the product of the series impedance, in Ω/m, and the shunt admittance, in S/m. Term C contains a, the *attenuation constant*, in nepers per meter, and term D contains β, the *phase constant,* in radians per meter. Comparing A with C and D, we see that:

$$\gamma = \alpha + j\beta$$

Eq 16

In term C, when a*x* = 1.0, the voltage *E* has attenuated by 1.0 neper to the factor $e^{-1.0}$ = 0.37. In other words, the exponent has the unit nepers. For example, if α=.0001 nepers per meter, then in 100 meters the attenuation is 0.01 neper. One neper is equivalent to 20 $\log_{10} e$ = 8.686 dB, so in this example the voltage attenuates 0.08686 dB. That is, in length *x* meters the reduction in voltage is:

$$\alpha(\text{nepers/m}) \bullet x(\text{m}) = \alpha x(\text{nepers}) = 8.686 \bullet \log_{10}(\alpha x)\ (\text{dB})$$

Eq 17

Since Z_0 is constant, this is also the reduction in power. In coax tables, the matched-line attenuation at some frequency is very often stated in dB per 100 feet. Multiplying this value by 0.032808 gives the dB per meter. Dividing this by 8.686 gives nepers per meter. So in *x* meters the attenuation is a*x* nepers. This is called the *matched loss*, ML, if the load is the same as Z_0 (no reflections and therefore no standing waves).

The D term is a unit vector that rotates clockwise as *x* increases. The phase at *x* *lags* the phase at *x*=0. When *b* *x*=1.0 the phase has shifted –1.0 radians (–57.3 degrees) and one complete revolution is -2π radians (–360 degrees). The total phase shift in radians is the radians per meter times the length in meters. The conversion from radians to degrees, or degrees to radians, is:

$$\text{degrees} = \text{radians} \bullet \frac{360}{2\pi}$$

$$\text{degrees} = \text{radians} \bullet \frac{360}{2\pi} \qquad \text{Eq 18}$$

A further comparison between part B and parts C and D leads to the following close approximation equations for *a* and *b*, for a low loss *coax* cable:

$$\alpha \approx \frac{R}{2Z_0} + \frac{GZ_0}{2} \quad \text{nepers per meter}$$

$$\beta \approx \omega\sqrt{LC} \approx \frac{f}{VF} \bullet 2.092 \bullet 10^{-8} \quad \text{radians per meter}$$

$$= \frac{2\pi}{\text{wavelength (meters)}} = \frac{2\pi f\,(\text{MHz})}{VF \bullet 299.79} \quad \text{radians per meter} \qquad \text{Eq 19}$$

If *G* is negligible, we can find *R,* in Ω/m, if the attenuation in nepers per meter is known. The formula for *b*, in radians per meter, is obtained by combining Eq 19 with Eqs 7, 9 and 21, and we see that *b* increases as frequency *f* (Hz) increases, and also as VF gets smaller.

Using the easily determined values of *a* and *b*, we can restate the low loss value for Z_0 that was given in Eq 5.

$$Z_0 \approx R_0\left(1 - j\frac{R}{\omega L}\right) = R_0\left(1 - j\frac{2\alpha R_0}{2\beta R_0}\right) = R_0\left(1 - j\frac{\alpha}{\beta}\right) \qquad \text{Eq 20}$$

Another frequently needed number is the wavelength, *l*, in meters or feet, of a length of coax, which is the free space wavelength times the velocity factor.

$$\lambda\,(\text{meters}) = \underbrace{\frac{2.998 \bullet 10^8\ (\text{m/s})}{f\,(\text{Hz})}}_{\text{free space wavelength } \lambda_0} \bullet VF$$

$$\lambda\,(\text{feet}) = \frac{983.6}{f(\text{MHz})} \bullet VF \qquad \text{Eq 21}$$

ARRL Radio Designer Program

Various handbooks such as the *ARRL Antenna Book*, the *ARRL Handbook* and the *ITT Reference Data for Radio Engineers* contain graphs of α, in dB per 100 feet, on a logarithmic vertical axis, versus frequency on a logarithmic horizontal axis, for a large number of coax types. On this kind of log-log graph the plots are approximately straight-line. Also, the transmission line models in simulation programs such as ARRL's *Radio Designer* (related to Compact's *Harmonica*) and others have options to include the frequency variation of a per unit length over a frequency range. In the two examples cited the form is:

$$\alpha\,(\text{dB / unit length}) = P\left(C1 \bullet \sqrt{f} + C2 \bullet f\right) \qquad \text{Eq 22}$$

where *P* is the number of unit lengths (meters, hundreds of feet, etc), *C*1 relates to resistance loss and *C*2 relates to dielectric loss. The values of *C*1 and *C*2 for a particular coax can be closely approximated by *curve-fitting* a line to the graph. The *Mathcad* graph plotting feature is very useful for this task. We advise the consistent use of meters of length.

The ARRL *Radio Designer* software is a valuable resource for analyzing transmission line circuits. It can be used to get frequency sweeps of complete systems, for example a transmatch, transmission line and a set of antenna impedance values. It can perform optimization and tuning operations of many kinds. [5] This program collaborates with *Mathcad*-type programs to give us some very powerful capabilities.

Wave Propagation (With Reflections)

If the load impedance Z_L is not equal to the complex conjugate of the complex Z_0, a portion of the *forward* power wave is said to be *reflected* from the load back toward the generator. An equivalent statement is that if the load impedance Z_L is not the complex conjugate of the complex Z_0, an impedance mismatch occurs and there is a *mismatch loss*, exactly the same as in lumped circuit theory, and the power that is accepted by the load is less than the power that is available. For the power wave, the difference between the forward power wave arriving *at* the load and the reflected power wave that is returned *from* the load is the power that is dissipated *in* the load, and it, plus the power that is lost in the line itself, is equal to the power that is *delivered* to the line by the generator. That is:

$$P_{generator} = \overbrace{\underbrace{P_{forward}}_{\text{at the load}} - \underbrace{P_{reflected}}_{\text{from the load}}}^{\text{in the load}} + P_{line\ loss} \qquad \text{Eq 23}$$

The phases of the forward and reflected voltage and current waves interact, either to enhance or diminish each other in a repetitive manner along the line. In order to limit the size of this article, we will deal mainly with the mathematical descriptions of this behavior, and avoid the lengthy word descriptions that are often used. The justification for this is that, these days, sophisticated and inexpensive math software, such as *Mathcad*, removes all of the pain from solving and *plotting* the difficult complex algebra equations so that the *visual* results are immediately available. Also, we will deal only with the rms values of steady-state sine-wave signals. With this approach, the discussion will be very clear and simple.

We are concerned with low-loss cable and mostly with voltage standing wave ratios (VSWR) that are not greater than, say 30 or so (VSWR will be discussed). The reasons for this will come later and this situation describes nearly all of the practical applications that we radio amateurs encounter in well-designed systems. In certain applications, though, such as short-circuit and open-circuit stubs or highly reactive loads, extremely high VSWRs are encountered and in fact the VSWR concept itself becomes impractical, especially when Z_0 becomes complex (see later discussion).

Standing Waves

Fig 4 shows a cable with length *l* and certain values of Z_0, α and β, connected between a generator and a complex load Z_L which is different from Z_0. The value *x* is the distance from the generator and *d* is distance from the load. Waves of voltage and current are traveling in both directions. Eqs 24 through 27 are the steady-state solutions to the wave equation for these bidirectional waves. We want to find the voltage, current, power level, impedance (or admittance) and VSWR at every point on the cable, over some frequency range and (often simultaneously) over some range of load values. We also want the power into the cable, the power into the load and the total cable loss. The procedure is to let the math program do the grunt work, solving and plotting the standard equations that we present. Our goal is to show a general approach that can be applied to many specific situations.

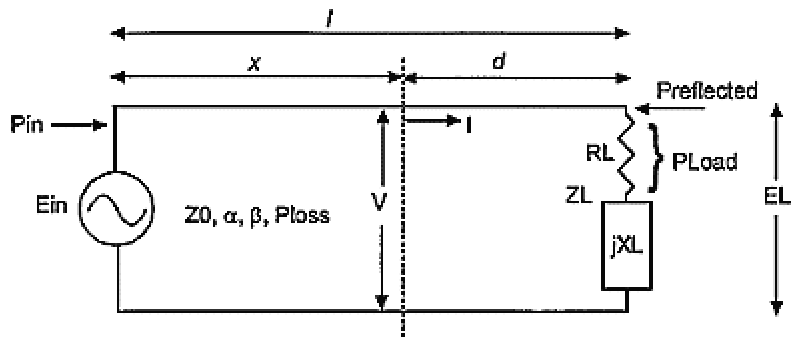


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Fig 4—Diagram of a cable and load showing the various parameters of the system.

To find the complex impedance at any distance *d* from the load, looking toward the load:

$$Z_d = Z_0 \frac{Z_L + Z_0 \tanh \gamma d}{Z_0 + Z_L \tanh \gamma d} \qquad \text{Eq 24}$$

where γ is found from previous equations, Z_0 and $\tanh \gamma d$ are complex, and if $d=l$, the length of the coax, then $Z_d = Z_{in}$, the input impedance. If Z_L is a short- or open-circuit, then from Eq 24:

$$Z_{sc} = Z_0 \tanh \gamma d \ \text{ (short ckt) or } \ Z_{oc} = \frac{Z_0}{\tanh \gamma d} \ \text{ (open ckt)}$$

$$Z_{sc} = Z_0 \tanh \gamma d \ \ \text{(short ckt)} \ \text{ or } \ Z_{oc} = \frac{Z_0}{\tanh \gamma d} \ \ \text{(open ckt)} \qquad \text{Eq 25}$$

If some reference value of input voltage E_{in} (eg, 1.0) is assumed, the voltage at some distance x (possibly l) from the generator is:

$$E_x = E_{in} \left(\cosh\ \gamma x - \frac{Z_0}{Z_{in}} \sinh\ \gamma x \right) \ \ \text{V} \qquad \text{Eq 26}$$

and the current is:

$$I_x = \frac{E_{in}}{Z_{in}} \left(\cosh\ \gamma x - \frac{Z_{in}}{Z_0} \sinh \gamma x \right) \ \ \text{A} \qquad \text{Eq 27}$$

Computer-generated plots of the magnitudes $|E_x|$ and $|I_x|$ of Eqs 26 and 27 display the *exact* standing-wave patterns for voltage and current. The power in the load is:

$$P_L = E_L \bullet E_L^* \bullet \text{Re}\left(\frac{1}{Z_L} \right) = |E_L|^2 G_L \ \ \text{W} \qquad \text{Eq 28}$$

where Re means the real part, E_L* is the complex conjugate of E_L*, Z_L is complex, $1/Z_L$ is the admittance $G+jB$ and the input power is:

$$P_{in} = E_{in} \bullet E_{in}^* \bullet \text{Re}\left(\frac{1}{Z_{in}} \right) = |E_{in}|^2 G_{in} \ \ \text{W} \qquad \text{Eq 29}$$

where E_{in} and Z_{in} are also complex. The cable loss in dB is:

$$P_{loss} = 10 \log\left(\frac{P_{in}}{P_L} \right) \ \ \text{dB} \qquad \text{Eq 30}$$

The voltage reflection coefficient, return loss and standing-wave ratio at the coax input are found as follows, with certain limitations that are discussed next:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad \text{complex voltage reflection coefficient}$$

$$\text{VSWR}_{in} = \frac{1 + |\Gamma_{in}|}{1 - |\Gamma_{in}|} \quad \text{voltage standing wave ratio}$$

$$\text{RL} = -20 \bullet \log_{10} |\Gamma_{in}| \quad \text{return loss, dB} \qquad \text{Eq 31}$$

Eqs 24 through 31 tell us the important things about the system, namely the input admittance of the coax (the transmatch has to deal with this), the power loss and voltages in the coax and, with some further analysis (beyond the scope of this article, but see Note 6), the voltage, current and power stresses in the transmatch (including any iron or ferrite transformers or inductors). The *input* VSWR is involved in interfacing the transmitter with the coax and must be reduced to a low number by a transmatch. [7] VSWR meters are usually calibrated to read values up to 3:1 or perhaps 10:1.

These numbers are also found for the open-circuit and short-circuit stubs and for pure reactance loads. For these situations the concepts of reflection coefficient and standing wave ratio are totally irrelevant but the real part of the input admittance enables us to find the power dissipated in the stub or line and the value of stub *Q*.

To explore this a little further, the concepts of reflection coefficient and VSWR, which are based on traveling waves of voltage or current, run into difficulty when Z_0 is not a pure resistance, in other words when the cable is *lossy* (see Eq 4). [8] For example, suppose that $Z_0 = 50 - j1.0$ and $Z_L = 50 + j1.0$. The line and load are then conjugately matched for maximum power transfer. But from Eq 31, the reflection coefficient is *j*0.02, the VSWR is 1.041 and RL=34 dB. For small values of reactance this error is trivial but for open-circuit or short-circuit stubs or highly reactive loads the errors get large and RL is essentially 0.0 dB. On the other hand, if we work with power values, as in Eqs 28, 29 and 30, the power numbers are correct and the usage of Γ and VSWR is avoided and unnecessary. Also, we find that the Z_0 used in Eqs 24-27 can be complex, with no resultant errors (see Eq 15, part B, where the values of *L*, *C*, *R* and *G* imply a complex Z_0 as defined in Eq 1). [9] In particular, Eq 26 for the load voltage is correct for a complex Z_0 and therefore provides the correct value of E_L in Eq 28. As a real-world matter, the error in VSWR is unimportant, nevertheless the use of the complex Z_0 produces better overall accuracy in the various calculations.

At this point, a brief remark about the transmatch might be interesting. Eq 23 says that the power delivered *by* the generator (minus the transmatch loss) is equal to the power that actually goes *into* the line. This power, minus the line loss, is *delivered* to the load. If the generator is *designed* to deliver a specified power level into a certain value of resistance, for example 50 Ω, then if the line's complex input impedance, found in Eq 24, can be *transformed* to this 50 Ω by a network of some kind, then the specified power will actually be delivered, *despite* any reflections from the load. The transmatch does this job. And we note also that in this transformation, no mention is made, or needed, of the generator's internal impedance. In fact, the generator impedance does not appear in *any* of our equations or discussions. Finally, a word about the load reflections: they cause the input impedance of the line to be different from Z_0 (the line is an impedance transformer), and when the transmatch transforms this input impedance to 50 Ω, the effects of these load reflections are completely accounted for and the specified power is delivered.

Note that the equations that we use in Eqs 24 through 30 are exact, and do not involve the approximations that have been used in the past to simplify manual calculations. This greatly reduces the array of possible choices in problem solving. This approach has been called the brute-force approach. This is no longer true with *Mathcad* and similar programs. We are limited only by the accuracies in the various parameters, which may involve precise lab measurements that are beyond the average amateur workshop or may be found from simulations of some kind. Reliable handbook data is often used instead.

Because of the measurement inaccuracies of load impedance, matched loss, velocity factor, cable length, etc, how accurately can we calculate system performance? The equations and simulations can be used to find the *sensitivities* to these errors. Impedance values are the most difficult to know without a good measurement (or truthful simulation) method and usually represent the biggest source of errors. Also, with a calibrated transmission line sufficiently long it is possible to measure inaccessible antenna impedances using the exact equations in this article—if we are careful to eliminate errors due to common-mode currents induced by the antenna.

Examples

To get a feel for how these equations work, the *Mathcad* worksheet of Sidebar A goes through a realistic example that we might run into in the ham shack. This example might be used as a template for a wide variety of similar exercises, including frequency sweeps, parameter sweeps and three-dimensional sweeps, that the reader can devise. In this example, meters of length, radians per meter and nepers per meter are used and are a good idea, in order to minimize confusion and errors. Matched loss, 0.57 dB per 100 feet, is converted to 0.002154 nepers per meter, a 50-foot coax length is converted to 50 × 0.3047 meters and the phase constant is in radians per meter. Coax power levels are in W, and we prefer the cable loss in dB. In this example, for a matched cable the loss would be 0.285 dB and with an input VSWR of 1.86 the loss is 0.337 dB. The additional loss is 0.052 dB. In other words, the standing waves of current and voltage produce greater losses in the copper and the dielectric, respectively. The load VSWR, if we want to know it, can be found by temporarily setting *A*, the dB per 100 feet, to 0.0. In this example the load VSWR is 1.92.

As a much more difficult example, consider at 28 MHz a 120 foot length of 50 – *j*2 Ω coax with 1.0 dB per 100-foot attenuation, connected to a 200 + *j*100 Ω load. Again, use meters, radians and nepers. Using Eq 24, plot $1/Z_d$ versus *d* (distance from load) to find the first point on the coax where the real part of the admittance (the conductance) is 1/50 S. Then use the imaginary part (the susceptance) of this $1/Z_d$ to calculate the length of a shorted stub (solve Eq 25 for *d*), using the same kind of coax, that has the negative value of susceptance and therefore produces a 50 +*j*0 Ω junction. Use Eq 24 again to find the coax input impedance at the generator (not *exactly* 50 Ω). Then, using Eqs 26, 28 and 29, find the actual and dB power loss in the stub, in the coax between the stub and the load and also between the stub and the generator, assuming the appropriate value of input voltage at the generator. Then plot input impedance (preferably admittance) versus frequency to see what the transmatch (if any) requirements are. This is a very messy problem using traditional methods, but is a lot easier using *Mathcad* or similar software. Once the worksheet has been set up, it becomes part of your library. For all of this work a comfort factor with math methods and some skill with the software (easily acquired) are necessary, though. Also, a familiarity with Smith Chart graphical methods and the ARRL *Microsmith* software can add extra dimensions to our understanding.

The equations in this article apply also to parallel-line transmission lines, except Eqs 6, 8, 9, 10 and 14, that apply to coax only. The references provide more details for parallel lines.

Acknowledgments

The problems associated with complex values of Z_0 with respect to the traveling voltage/current wave reflection coefficient and VSWR were realized many years ago (as shown in Notes 8 and 9). I appreciates the efforts of Dean Straw, N6BV, (at ARRL) and Frank Witt, AI1H, in bringing this to his attention, and also their comments and suggestions for this article (but any mistakes belong to me). Also, Dean's excellent TL.EXE program has been very helpful in writing this article and comparing the numbers.

Finally, the ITT handbook *Reference Data for Radio Engineers*, Transmission Lines chapter, is a gold mine of data, formulas and information (some of which can be rather intimidating) that can be used to adapt and modify the equations and procedures of this article to other applications, especially twin-lead and ladder-line, which are popular among hams. As mentioned before, many of the special-case formulas and simplified formulas are not needed by *Mathcad*. The ARRL *Antenna Book* is also highly recommended.

Notes

[1]Sabin, W. E., W0IYH, "**Mathcad 6.0: A Tool for the Amateur Experimenter**," *QST*, April 1996, p 44.

[2]Kraus, J., W8JK, *Elecromagnetics*, Fourth Edition, p 579, McGraw-Hill NY, 1994.

[3]Brainard, WA1ZRS, and Smith (Times Wire and Cable Co), "Coaxial Cable—the Neglected Link," *QST* April 1981, p 28.

[4]Ferber, M., W1GKX, (Times Wire and Cable Co), "Coaxial Cable Attenuation," *QST,* April 1959, p 20.

[5]Sabin, W. E., W0IYH, "Broadband HF Antenna Matching With ARRL *Radio Designer,*" *QST*, August 1995, p 33.

[6]Witt, Frank, AI1H, "How to Evaluate Your Antenna Tuner," Part 1, *QST,* April 1995, p 30; Part 2, May 1995, p 33.

[7]Sabin, W. E., W0IYH, "The Lumped-Element Directional Coupler," *QEX*, March 1995 p 3, and May 1995 p 8.

[8]Kurokawa, K., "Power Waves and the Scattering Matrix," *IEEE Transactions on Microwave Theory and Techniques*, March 1965, Sec IX, p 201.

[9]Johnson, W. C., *Transmission Lines and Networks*, Chapter 4, pp 93-96. McGraw-Hill, New York, 1950.

Calculate an appropriate cable size. A 3-phase sub-main circuit having a design fundamental current of 100A is to be wired with 4/C PVC/SWA/PVC cable on a dedicated cable tray. Assuming an ambient temperature of 30Â°C and a circuit length of 40m, calculate an appropriate cable size for the following conditions: CASE 1 // Undistorted balanced condition using traditional method (cosÎ¸ = 0.85). CASE 2 // Undistorted balanced condition with a max. copper loss of 1.5% (cosÎ¸ = 0.85).