Course Summary for "Singularities, Black Holes and Thermodynamics in Relativistic Spacetimes"

Dr. Erik Curiel
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http://strangebeautiful.com/lmu/2014-summer-sings-bhs-thermo.html

Summer, 2014
Mo. 12:00–14:00 C.T.
Ludwigstr. 31, 021

1 Course Description

In this course, we will investigate the nature of singularities and black holes in general relativity, and their possible relation to thermodynamics, through a rigorous examination of their mathematical representation in the theory. Except for the section on Hawking radiation, we will treat all topics at the purely classical (*i.e.*, non-quantum level). The particular topics we will cover are: global and local causal structure of spacetimes; the nature and existence of singularities; the laws of black-hole mechanics and their relation to thermodynamics; and the nature of cosmological singularities. A basic working knowledge of general relativity and differential geometry is assumed (*e.g.*, the sort one would get in an advanced undergraduate or introductory graduate course). I will quickly review some elements of differential geometry and of general relativity itself at the beginning of the course, but that treatment will in no way suffice for one who does not already know something about the subject.

The philosophical interest of the subject spans several fields. With regard to the study of causal structure, questions we may address include the physical meaning and the physical possibility of different kinds of causal pathology and their bearing on issues such as the possibility of time-travel of some sort, and whether general relativity is a deterministic theory or not, and what the possibilities are for observation and prediction of global structures. General relativity's prediction that singularities occur generically in spacetimes under seemingly physically reasonable conditions poses several important questions. How does one define a singularity, and is there a preferred canonical definition? Does their prediction herald the breakdown of the theory, or do singularities really occur in the actual universe? If they do occur, what exactly are they? Can one attribute existence in any sense to them? The issues pertaining to black holes range from the nature of inter-theoretic relations (*e.g.*, what is the meaning of "entropy" when applied to black holes? is it a true thermodynamical entropy?) to the nature and direction of the arrow of time, to issues of determinism and predictability.

2 Evaluation

The course is worth 9 ECTS. The grade for the course will be determined by a 16–18 page term paper due September 19. The paper will be on a subject of the student's choice (more physics or

more philosophy, as the student likes); I strongly urge students to consult with me before choosing a topic, however. I will assign 7 sets of homework problems over the course of the term, and I strongly encourage students to complete them, but they will not count towards final evaluation of the grade.
Students wishing to audit the course should consult with me prior to or during the first week of classes.

3 Readings

The two required texts for the course, Wald (1984), *General Relativity*, and Malament (2012), *Topics in the Foundations of General Relativity and Newtonian Gravitational Theory*, are available at any good academic bookstore or online book seller. (The uncorrected proofs of Malament's book are also available for free download as a PDF; see the reference in the bibliography.) Other required or suggested works should be on reserve at the university library or available online as cited. Many of the required and suggested readings are available online at the course's website, `http://strangebeautiful.com/lmu/2014-summer-sings-bhs-thermo.html`, though they may not be listed as such in the bibliography.

Although not required, I strongly urge students also to acquire Hawking and Ellis (1973), *The Large Scale Structure of Space-Time*, and Wald (1994), *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*.

4 Tentative Schedule

DIFFERENTIAL GEOMETRY AND GENERAL RELATIVITY

Week 1 (Apr. 07) introduction to general relativity; differential manifolds

Week 2 (Apr. 14) tangent vectors; vector fields; integral curves; cotangent vectors

Week 3 (Apr. 21) HOLIDAY: NO LECTURE

Week 4 (Apr. 28) abstract-index notation; tensor analysis; diffeomorphisms; action of diffeomorphisms on tensors

Week 5 (May 05) the Lie derivative; derivative operators; geodesics; curvature; (pseudo-)Riemannian metrics

Week 6 (May 12) relativistic spacetimes; stress-energy and the Einstein field-equation; Killing fields (isometries); conserved quantities

CAUSAL STRUCTURE

Week 7 (May 19) orientability; causality conditions and closed timelike curves; domains of dependence and causal horizons; global hyperbolicity

SINGULARITIES

Week 8 (May 26) geodesic congruences and the Raychaudhuri equation; conjugate points; incomplete curves; extensions of spacetimes

Week 9 (Jun. 02) possible definitions of a singularity; the Geroch-Hawking-Penrose theorems; naked singularities and cosmic censorship

Week 10 (Jun. 09) HOLIDAY: NO LECTURE; my birthday!

BLACK HOLES AND THERMODYNAMICS

Week 11 (Jun. 16) sphericity, staticity, stationarity, axisymmetry; derivation of the Kerr and Schwarzschild metrics; Birkhoff's Theorem; Lense-Thirring effect; Penrose process; conformal infinity; trapped surfaces and event horizons; asymptotically flat black holes; uniqueness theorems

Week 12 (Jun. 23) the four laws of black-hole mechanics; the thermodynamical character of black holes

Week 13 (Jun. 30) Hawking radiation; black-hole evaporation; information loss; non-unitary evolution

COSMOLOGICAL SINGULARITIES AND THERMODYNAMICS

Week 14 (Jul. 07) homogeneity and isotropy; derivation of the FLRW spacetimes and their properties; dark energy and the cosmological constant; the initial-state problem; Penrose's Weyl Curvature Hypothesis; sudden singularities; possible measures of gravitational entropy

TERM PAPER DUE

Sep. 19

References

Hawking, S. and G. Ellis (1973). *The Large Scale Structure of Space-Time*. Cambridge: Cambridge University Press.

Malament, D. (2012). *Topics in the Foundations of General Relativity and Newtonian Gravitational Theory*. Chicago: University of Chicago Press. Uncorrected final proofs for the book are available for download at http://strangebeautiful.com/other-texts/malament-founds-gr-ngt.pdf.

Wald, R. (1984). *General Relativity*. Chicago: University of Chicago Press.

Wald, R. (1994). *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. Chicago: University of Chicago Press.

Thermodynamics in Relativistic Spacetimesâ€. Dr. Erik Curiel Erik.Curiel@lrz.uni-muenchen.de. oce: Ludwigstr.Â In this course, we will investigate the nature of singularities and black holes in general relativity, and their possible relation to thermodynamics, through a rigorous examination of their mathematical representation in the theory. Except for the section on Hawking radiation, we will treat all topics at the purely classical (i.e., non-quantum level). The particular topics we will cover are: global and local causal structure of spacetimes; the nature and existence of singularities; the laws of black-hole mechanics and their relation to thermodynamics; and the nature of cosmological singulariti... Singularities and Black Holes. A singularity of a function is a limit at which the function is ill-defined â€" typically because of a discontinuity or infinity entering into the equation. For example, the function f=1/x is singular at x = 0. In the context of spacetime theory, singularities are limits (or, loosely speaking, "regions") in which the Einstein field equations break down.Â Taking relativistic considerations into account, however, we find that black holes are far more exotic entities.Â Further, mass in black hole mechanics is mirrored by energy in thermodynamics, and we know from relativity theory that mass and energy are actually equivalent. Connecting the two sets also requires linking the surface area of a black hole with entropy, as Bekenstein had suggested. A gravitational singularity, spacetime singularity or simply singularity is a location in spacetime where the mass and gravitational field of a celestial body is predicted to become infinite by general relativity in a way that does not depend on the coordinate system. The quantities used to measure gravitational field strength are the scalar invariant curvatures of spacetime, which includes a measure of the density of matter. Since such quantities become infinite at the singularity, the laws of normal