# Analysis on recurrence properties of Weyl's curvature tensor and its Newtonian limit

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**Abstract.** In this note we present recent enhancements in the cosmological literature related to H.Weyl's conformal curvature tensor. We outline generalized recurrence properties of this tensor in Weyl's manifold and then delineate its Newtonian limit. As well, we discuss some relativistic equations under Newtonian limit, which shall illustrate the role of electricity and magnetism in the geometry of space-time.

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## 1 Introduction

Hermann Klaus Hugo Weyl (9 November 1885-8 December 1955) is one of the most influential German Mathematicians of $19^{th}$ century, who dedicated most of his life in developing an enthusiastic geometry now universally known as *geometry of Weyl's space*. Weyl published technical and general works on space, time, matter, philosophy, logic symmetry and history of Mathematics. He was one of the first who visualized general relativity with the laws of electromagnetism. H. Weyl commented that ([17]) *"my work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful"* sums up his efficiency, perfection and curiosity, not only in Mathematics, but in Physics of space-time, matter and Philosophy also. In search of "what is the role of electricity in the geometry of space-time"? Weyl concentrated to this topic in paper after paper and book after book. The standpoints that he and his successors opened up are probably explored today, under the names of "gauge field theory" and "grand unified field theory," by more investigators than there were in the entire physics community at the time of Weyl's first paper in the field. Weyl invented the gauge concept in 1918. By 1928 he had reformulated and restated the idea in the way it is still understood today: "gauge invariance corresponds to the conservation of charge as a coordinate invariance corresponds to the conservation of energy-and-momentum". In a 1918 article, Hermann Weyl ([16, p. 465]) tried to combine electromagnetism and gravity by requiring the theory to be invariant under a local scale change of the metric, i.e., $g_{uv} \rightarrow g_{uv}e^{\alpha(\chi)}$, where $\chi$ is a 4-vector.

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This attempt was successful and was characterized by Einstein for being inconsistent with observed physical results. It predicted that a vector parallel transported from point $p$ to $q$ would have a length that was path dependent. Similarly, the time interval between ticks of a clock would also depend on the path along which the clock was transported. The article did, however, bears the following crucial facts:

a) The original term for "Gauge invariance" was *"Erich-invarianz"*, which Weyl used in his article. This term refers to invariance under Weyl's scale change.

b) The geometric interpretation of electromagnetism.

c) The beginning of non-Abelian Gauge theory. The similarity of Weyl's theory to non-Abelian Gauge theory is more striking in his 1929 paper[1].

Now, in order to pursue our proposed study "A review on recurrence properties of Weyl's curvature tensor and its Newtonian limit", we briefly introduce some notions on Weyl's space, Generalized Weyl's space and Newtonian limit of general relativity.

**Weyl's space**

An $n-$dimensional differentiable manifold $W_n$ is said to a Weyl space, if it has a symmetric connection $\nabla$ and a symmetric conformal metric tensor $g_{ij}$ preserved by $\nabla$ satisfying the compatibility condition given by the equation ([10],[12],[15])

$$\nabla_k g_{ij} - 2T_k g_{ij} = 0, \tag{1.1}$$

where $T_k$ represents a covariant vector field.
Re-writing (1.1) in an extended fashion, we have

$$\frac{\partial}{\partial x^k} g_{ij} - g_{hj}\Gamma^h_{jk} - 2T_k g_{ij} = 0, \tag{1.2}$$

where $\Gamma^i_{kl}$ are the connection coefficients of the symmetric connection $\nabla$ and are defined as

$$\Gamma^i_{kl} = \begin{pmatrix} i \\ k\ l \end{pmatrix} - g^{im}\left(g_{mk}T_l + g_{ml}T_k - g_{kl}T_m\right). \tag{1.3}$$

Moreover, under the renormalization condition:

$$\tilde{g}_{ij} = \lambda^2 g_{ij}, \tag{1.4}$$

of the metric tensor $g_{ij}$, the covariant vector field $T_k$ is transformed by the law:

$$\tilde{T}_k = T_k + \frac{\partial}{\partial x^k}\ln\lambda, \tag{1.5}$$

where $\lambda$ is a scalar function defined on $W_n$. Thus the space $W_n$ satisfying all the foregoing condition will be symbolized by $W_n(\Gamma^i_{jk}, g_{ij}, T_k)$, or $W_n(g,T)$. Also, a geometric object $\Omega$ defined on $W_n(\Gamma^i_{jk}, g_{ij}, T_k)$ is called a satellite of weight $\{w\}$, of a tensor $g_{ij}$, if it admits a transformation of the form:

$$\tilde{\Omega} = \lambda^w \Omega \tag{1.6}$$

[1]Weyl, H., Proc. Natl. Acad. Sci. 15 (1929), 32.

under renormalization condition of the metric tensor $g_{ij}$ ([10],[5]).
Further, the prolonged covariant derivative of a satellite $\Omega$ is defined by:

$$\dot{\nabla}_k \Omega = \nabla_k \Omega - w T_k \Omega. \tag{1.7}$$

It is remarkable that the prolonged derivative preserves the weight.

**Generalized Weyl's space**

An $n-$dimensional differentiable manifold $GW_n$ having an anti-symmetric connection $\nabla^*$ and anti-symmetric metric tensor $g^*_{ij}$ preserved by $\nabla^*$ is called a "generalized Weyl space" [9]. For such a space, in local co-ordinate system, we have a compatibility condition as below:

$$\nabla^*_k g^*_{ij} - 2T^*_k g^*_{ij} = 0, \tag{1.8}$$

where $T^*_k$ are the components of a covariant vector filed called the complementary vector field of the $GW_n$ space. Using the concept of covariant differentiation ([18],[8]), the compatibility condition, (1.8) can be written in extended form as:

$$\frac{\partial^*}{\partial x^k} g^*_{ij} - g^*_{hj} L^h_{ik} - g^*_{ih} L^h_{jk} - 2T^*_k g^*_{ij} = 0, \tag{1.9}$$

where $L^i_{kl}$ are the connection coefficients of the anti-symmetric connection $\nabla^*_k$ and are obtained from the compatibility condition as [1]:

$$L^i_{kl} = \Gamma^i_{kl} + \frac{1}{2}\left[\chi^h_{km} g^*_{lh} + \chi^h_{ml} g^*_{kh} + \chi^h_{kl} g^*_{hm}\right] g^{*mi}. \tag{1.10}$$

Now, putting

$$\chi^i_{kl} = \frac{1}{2}\left[\chi^h_{km} g^*_{lh} + \chi^h_{ml} g^*_{kh} + \chi^h_{kl} g^*_{hm}\right] g^{*mi}, \tag{1.11}$$

we obtain

$$L^i_{kl} = \Gamma^i_{kl} + \chi^i_{kl}, \tag{1.12}$$

where $\Gamma^i_{kl}$ and $\chi^i_{kl}$ are respectively the coefficients of a Weyl connection and the torsion tensor of $GW_n$ space and are expressed as:

$$\Gamma^i_{kl} = \frac{1}{2}\left(L^i_{kl} + L^i_{lk}\right) = L^i_{(kl)}, \tag{1.13}$$

(here, the round bracket stands for symmetry.) and

$$\chi^i_{kl} = \frac{1}{2}\left(L^i_{kl} - L^i_{lk}\right) = L^i_{[kl]} \tag{1.14}$$

(here the square bracket stands for anti-symmetry).
A generalized Weyl space satisfying all the aforementioned conditions is symbolized as $GW_n\left(L^i_{jk}, g^*_{ij}, T^*_k\right)$.

**Newtonian limit in General Relativity**

Some researchers attempted to establish a closed Newtonian system for the evolution of fluid quantities (for instance, density, expansion, shear etc.) by considering Newtonian limit of the corresponding equation in general relativity. Afterward, a considerable amount of research has been dedicated towards the supporting or disproving this goal by [6] and many other researchers. Actually, the concept of Newtonian limit in theory of relativity has been introduced to focus on two major aspects; one of them concerns to the presentation of a precise derivation of Newtonian limit of fluid evolution equation in a $4-$dimensional "frame theory" developed by ([3], [2]). This theory covers both Einstein and Newtonian theory of gravitationally interacting matter. On the other hand, second aspect pertains to the discussion general relativistic equations to describe a closed Newtonian system.

## 2 Recurrence properties of Weyl's curvature tensor in $W_n$ and $GW_n$ spaces

According to [10], under a renormalization condition of the fundamental metric tensor $g_{ij}$ of the form (1.4), an object $\Omega$ defined on $GW_n$ space, admits a transformation of the form (1.6) is called a satellite with weight $\{w\}$ of the metric tensor and the prolonged covariant derivative of the satellite $\Omega$ relative to the symmetric connection $\nabla$ is defined by [11]:

$$\dot{\nabla}_k \Omega = \nabla_k \Omega - wT_k \Omega. \tag{2.1}$$

Whereas, the same relative to anti-symmetric connection $\nabla^*$ is defined as:

$$\dot{\nabla}^*_k \Omega = \nabla^*_k \Omega - wT^*_k \Omega, \tag{2.2}$$

which evinces that the prolonged derivative preserves the weight of satellite.
Now, the conformal Weyl curvature tensor $C^i_{jkl}$ of the generalized Weyl space $GW_n$ is given by [7] as:

(2.3)
$$C^i_{jkl} = R^i_{jkl} + \frac{2}{n(n-2)}\left(\delta^i_l R_{[jk]} - \delta^i_k R_{[jl]} - g_{jl}g^{im}R_{[mk]} + g_{jk}g^{im}R_{[ml]} - (n-2)\delta^i_j R_{[lk]}\right) -$$
$$-\frac{1}{(n-2)}\left(\delta^i_l R_{jk} - \delta^i_k R_{jl} - g_{jl}g^{im}R_{mk} + g_{jk}g^{im}R_{ml}\right) + \frac{R}{(n-1)(n-2)}\left(g_{jk}\delta^i_l - g_{jl}\delta^i_k\right),$$

where the square bracket stands for the anti-symmetrization.
This $n-$dimensional $GW_n$ space is said to be conformally recurrent Weyl space if its conformal curvature tensor (2.3) of weight $\{0\}$, on taking the prolonged derivative, satisfies the condition:

$$\dot{\nabla}^*_m C^i_{jkl} = \phi_m C^i_{jkl}, \tag{2.4}$$

where $\phi_m \neq T_m$ is a non-zero covariant recurrence vector field of weight $\{0\}$.
The conformal curvature tensor given by (2.3) can be re-defined in the purely contravariant pattern as below:

$$C^{ijkl} \equiv R^{ijkl} - \frac{1}{(n-2)}\left(g^{il}S^{kj} - g^{il}S^{jk} - g^{jk}S^{li} + g^{jl}S^{ki}\right), \tag{2.5}$$

where $S^{lj} \equiv R^{lj} - \frac{1}{2(n-1)} g^{lj} R$. $R^{lj}$ and $R$ are the Ricci quantities, while $C^{ijkl}$ bears the following properties:
(a). It is purely covariant against conformal redefinition of the metric, i.e., $g_{kl}(x) \rightarrow \lambda(x) g_{kl}(x)$.
(b). It vanishes if and only if the $GW_n$ space is conformally flat, i.e., $g_{kl}$ is diffeomorphic to $\lambda \eta_{kl}$, where $\eta_{kl}$ is flat.
(c). It possesses the symmetries of Riemannian tensor and also is traceless in each index pair.
(d). Evidently, from the properties (a) and (b), the Weyl tensor acts as a template for conformal flatness. Thus by evaluating it on a specific metric tensor, one can distinguish, whether the space-time under consideration is conformally flat or not.
In three dimensions the Weyl tensor vanishes identically and the Riemannian tensor is given by the last term in (2.5) at ($n = 3$). But not all three dimensional space-times are conformally flat.
Now, as it is well known that in dimensions greater than three, the conformal tensor (2.3) and (2.5) is the Weyl tensor, then what about the three dimensional space-time? To overcome from this difficulty, one need a substitute for Weyl tensor, which would act as a template for the conformal flatness. Indeed, there is a crucial substitution for Weyl tensor, known as *Cotton Tensor*, which is delineated as:

$$C^{ij} = \frac{1}{2\sqrt{g}} \left( \epsilon^{ikl} D_k R_l^j + \epsilon^{jkl} D_k R_l^i \right). \tag{2.6}$$

This Cotton tensor serves that role, as it possesses conformal template properties (a) and (b). $C^{ij}$ is symmetric in its indices and like the Weyl tensor, it is traceless.
Therefore, in case of the space-time having dimension less than 4, one can have the conformal recurrence properties for such conformally non-flat continuum by taking the prolonged derivative of Cotton tensor (2.6) of weight $\{0\}$. The conformally non-flat space-time having dimension $n = 3$ will be called conformally recurrent if the Cotton tensor satisfy the following:

$$\dot{\nabla}^*_m C^{ij} = \phi_m C^{ij}, \tag{2.7}$$

where $\phi_m$ is a non-zero recurrence vector field of weight zero.
It is remarkable that for the space-time having dimension less than three, there is no need of a conformal tensor and indeed none exists as all such spaces are locally conformally flat.

We, now, study conformal Weyl tensor, its Newtonian limit and some relativistic equations in general relativity due to Newtonian limit.

## 3 Conformal Weyl's curvature tensor, its Newtonian limit and relativistic equations

In the modern cosmological literature, role of conformal Weyl's curvature tensor has been adequately discussed. Especially, concentration upon the crucial parts, namely "electric" and "magnetic" parts of conformal Weyl's curvature tensor has been drawn by ([3], [2]) individually. Further, ([3], [2]) have discussed some ideas on the magnetic part of Weyl tensor under Newtonian limit of general relativity. In order to pursue significance of Weyl tensor in cosmological structures, the "frame-theory" for a general 4−dimensional space-time continuum is employed, which encapsulate both the "Newtonian's theory" as well as "Einstein's theory". In this frame theory, a single parameter $\varepsilon = c^2$ is introduced in such a way that it distinguishes between Newtonian's and Einstein's theories. The limit for this parameter is taken as $\varepsilon \to 0$. Moreover, in the formalism of Einstein's theory, a temporal metric $t_{ij}$ and an inverse spatial metric $s^{ij}$ are used which are related by an expression of the form:

$$t_{ij}S^{jk} = -\varepsilon\delta_i^k. \tag{3.1}$$

In case of general relativity, the parameter $\varepsilon$ is taken to be greater than zero, i.e., $\varepsilon > 0$ and the Riemannian as well as inverse Riemannian metric $g_{ij}$ and $g^{ij}$ are used such that:

$$g_{ij} = -\varepsilon^{-1}t_{ij} \text{ and } g^{ij} = S^{ij}. \tag{3.2}$$

On the other hand, in Newtonian's theory, the parameter $\varepsilon$ is taken to be zero and the temporal metric is defined as:

$$t_{ij} = t_{,i}t_{,j}, \tag{3.3}$$

where $t$ is the absolute time and the comma $(,)$ denotes the derivative with respect to the Eulerian co-ordinates, which will be discussed further.
In the conception of frame theory, the conformal Weyl's curvature tensor (2.3), for $\varepsilon > 0$ takes the following form:

$$C^i_{jkl} = R^i_{jkl} - \delta^i_k R_{lj} - \varepsilon^{-1}\left[t_{jk}R_{lm}s^{mi} + \frac{1}{3}\delta^i_k t_{lj}R_{pr}z^{pr}\right]. \tag{3.4}$$

This expression becomes worthless for $\varepsilon = 0$. However, if one uses the Einstein's field equation:

$$R_{ij} = 8\pi G\left(t_{ik}t_{jl} - \frac{1}{2}t_{ij}t_{kl}\right)T^{kl} - \Lambda t_{ij} \tag{3.5}$$

of the frame theory (valid for $\varepsilon \geq 0$), to eliminate $R_{ij}$ from the equation (3.4), we obtain:

$$C^i_{jkl} = R^i_{jkl} - 8\pi G\left[\delta^i_k t_{lp}t_{jr}T^{pr} - t_{jk}t_{lm}T^{mi} - \frac{2}{3}\delta^i_k t_{lj}t_{pr}T^{pr}\right]. \tag{3.6}$$

Now, this formula is noteworthy, even for $\varepsilon = 0$. Thereby, one can define the conformal Weyl's curvature tensor in the frame theory by equation (3.6). This expression for the Weyl's curvature tensor is quite suitable, for instance, if a sequence of general relativistic solutions has a Newtonian solution as a limit, then the limit of conformal Weyl curvature tensor is surely produced by (3.6). Furthermore, in case of Newtonian's theory, ($\varepsilon = 0$), equation (3.6) due to equation (3.3) reduces to the form:

$$C^i_{jkl} = R^i_{jkl} - \frac{8\pi G}{3}\rho t_{,j}\delta^i_k t_{,l}. \tag{3.7}$$

Also, the "electric" and "magnetic" part of the conformal Weyl curvature tensor with respect to any $4-$velocity vector $v^i$ can be obtained from (3.7) and these are respectively defined like below:

$$E^i_k = R^i_{kjl}v^j v^l - \frac{4\pi G}{3}\rho\left(\delta^i_k - v^i t_{,k}\right); \tag{3.8}$$

$$H_{ik} = \frac{1}{2}\eta_{ijpr}s^{ru}C^p_{ukl}v^j v^l = 0. \tag{3.9}$$

Here it is very observable that the magnetic part $H_{ik}$ of the conformal Weyl tensor vanishes in the Newtonian limit. This fact can be more precisely justified as follows:

In the general relativity, $H_{ik}$ measures the relative rotation of nearby freely falling gyroscopes due to gravitomagnetism. This effect has nonexistence in case of Newtonian theory in which the parallelism of spatial vectors is path independent. It means, parallel gyroscopes will always remain parallel if subjected to nothing except inertia and gravity.

Here, we now discuss some relativistic equations in Newtonian limit.

In three dimensional Weyl space, the equation (3.8) yields a new kind of tensorial quantity called "Newtonian tidal tensor" which is trace free part of gravitational field tensor $g_{(i,j)}$ (here comma denotes derivative with respect to Eulerian co-ordinate system) and is given by

$$E_{ij} = g_{i,j} - \frac{1}{3}\delta_{ij}g_{l,l}, \tag{3.10}$$

provided

$$E_{[ji]} = 0;\ E_{ii} = 0. \tag{3.11}$$

Likewise any Eulerian field, the Newtonian tidal tensor of the gravitational field strength $\vec{g}$ can be written in terms of Lagrangian co-ordinates as below:

$$E_{ij} = g_{i|k}J^{-1}_{kj} - \frac{1}{3}\delta_{ij}g_{l|k}J^{-1}_{kl}, \tag{3.12}$$

where a vertical slash stands for the derivative with regards to Lagrangian co-ordinate system.

In order to discuss evolution of tidal tensor as relativistic equation, we introduce a diffeomorphism $\vec{f}_t : \vec{x} = \vec{f}\left(\vec{X}, t\right)$, which sends fluid elements from their initial Lagrangian position $\vec{X}$ to a point $\vec{x}$ in the Eulerian space at time $t$. Also, we use an expression for the Jacobian of the inverse transformation:

$$\vec{h} = \vec{f}^{-1},\ \vec{g}[\vec{x}, t] = \ddot{\vec{f}}\left(\vec{h}[\vec{x}, t], t\right),\ J = \det\left(f_{i|k}\right), \tag{3.13}$$

and

$$h_{j|l} = \frac{1}{2J}\varepsilon_{jpq}\tau\left(\ddot{f}_l, f_p, f_q\right) - \frac{1}{3}\varepsilon_{opq}\tau\left(\ddot{f}_o, f_p, f_q\right)\delta_{ij}, \tag{3.14}$$

so that the Newtonian tidal tensor could explicitly be expressed in terms of $\overrightarrow{f}$ as:

$$E_{ij} = \frac{1}{2J}\left[\varepsilon_{jpq}\tau\left(\ddot{f}_l, f_p, f_q\right) - \frac{1}{3}\varepsilon_{opq}\tau\left(\ddot{f}_o, f_p, f_q\right)\delta_{ij}\right]. \tag{3.15}$$

Therefore, any trajectory field $\overrightarrow{f}$, which obeys the Lagrange-Newtonian system is given as:

$$\tau\left(\ddot{f}_J, f_j, f_k\right) = 0, \tag{3.16}$$

and

$$\tau\left(\ddot{f}_1, f_2, f_3\right) + \tau\left(\ddot{f}_2, f_3, f_1\right) + \tau\left(\ddot{f}_3, f_1, f_2\right) - \Lambda J = -4\pi G \overset{0}{\rho}, \tag{3.17}$$

where $\Lambda$ is a cosmological constant.
The last two equations determine the evolution of tidal tensor in the form of relativistic equation of general relativity via equation (3.12). In equations (3.16) and (3.17), the symbol $\tau(A, B, C)$ etc. denote the functional determinant of any functions $A(\vec{X}, t)$, $B(\vec{X}, t)$ and $C(\vec{X}, t)$ with respect to Lagrangian co-ordinate system $(\vec{X})$ and $\overset{0}{\rho}$ denotes the initial density field [4].

## 4 Concluding remarks

In the present manuscript, we have tried to draw our focus on recurrence properties of conformal Weyl's curvature tensor and its applications in the modern literature of relativity and cosmology. Particularly, we have discussed Weyl space, generalized Weyl space and their conformal curvature tensors along with recurrent nature. It is shown that for the space-time having dimensions less than 4, needed a tensor (called Cotton tensor), other than Weyl's tensor to check out the conformal flatness of the space-time and its recurrent nature. Moreover, a relativistic form of Weyl's tensor and relativistic equation evolved due to its parts (namely, electric and magnetic) has been studied.

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Properties of second-order tensors. Tensor diagonalization. The matrix representation of a tensor becomes especially simple in a basis made of eigen-vectors, when there is one.Â  First order degenerate points, with major and minor separatrices and tensor lines [Kratz].Â Visualization and analysis of second-order tensors: Moving beyond the symmetric positive-denite case. In Computer Graphics Forum, volume 32, pages 49-74. Wiley Online Library, 2013.