WG2, Topic #8: Different conceptions of the mathematical knowledge needed for teaching and how it can be acquired, and its relation to pedagogical knowledge

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Erkki Pehkonen
University of Helsinki
Finland

Problem solving in mathematics education in Finland

Abstract
The main content of the paper is to describe problem solving in Finnish school mathematics, since this is the picture of mathematics teaching that is convoyed to teacher students at universities. The description begins with considering Finnish mathematics curricula with the focus on the role of problem solving. Furthermore, different manifestations of problem solving in mathematics textbooks are discussed as well as how Finnish teachers implement problem solving in mathematics lessons. Additionally the way teachers use problem solving in assessment is discussed briefly. At the end of the paper, a new solution for teaching problem solving within the curriculum is dealt with. Such a reform is based on the use of problem solving as a teaching method that often is manifested by the use of open problems.

The purpose of school education in each country is, more or less, to develop independent, self-confident, critically thinking, motivated and multitalented individuals who will manage in different societal settings that they will encounter later on in their life. The key question is what kind of school instruction is optimal for this goal.

Problem solving in Finnish school mathematics

Problem solving has generally been accepted as a means for advancing thinking skills (e.g. Schoenfeld 1985). For example, in the NCTM Standards it is stated: *"Solving problems is not only a goal of learning mathematics but also a major means of doing so. … In everyday life and in the workplace, being a good problem solver can lead to great advantages. … Problem solving is an integral part of all mathematics learning."* (NCTM, 2000, 52)

Here we will not go into conceptual problems, but to point out to the existing literature on the topic (e.g. Pehkonen 2004). But for a paper trying to describe the implementation of problem solving in one particular country, it is important to explain the conception of problem solving within that country or at least its author's conception. We will adopt the following characterization of a problem (e.g. Kantowski 1980), which is widely used in the literature and rather much used in Finland: A task is said to be a *problem* if its solution requires that an individual combines previously known data in a way that is new (to him). If he can immediately recognize the measures that are needed to complete the task, it is a *routine task* (or a standard task or an exercise) for him. Furthermore, *problem solving* can be understood as "*a process where previously acquired data are used in a new and unknown situation*" (NCSM 1989).

Often in the beginning of problem solving, pupils deal with problems where they need only to have one insight in order to find the solution. Usually the key point is to perceive the problem situation in a new way. Such problems are called *one-step problems* or mathematically simple problems; the wording *mathematical puzzle* is also used. For example, matchstick problems are usually such. In the 1970's the term 'investigation' was introduced

in England to mean an extended problem situation. In an investigation the starting situation is usually given and, perhaps, the first problems, too, in order to show some possible ways ahead. Pupils are expected to choose their problems and route. Many examples of problems, mathematical puzzles and investigations can be found e.g. in the published paper Pehkonen (2004).

Problem solving in the Finnish curriculum. Curriculum development in Finland has reflected the international trends – usually with a delay of about 10 years. After the "new math" movement in the late 1960's and early 1970's, there was a shift 'back to basics' towards the end of the decade. Since the 1980's a lot of emphasis has been given for problem solving. (Kupari 1999) For more than twenty years, problem solving has been one of the general overall goals in the Finnish curricula (NBE 1985, 1994, 2004). Its implementation has been in the focus of teacher pre-service and in-service education since the end of the 1980's.

In 1986 the National Board of Education made systematic efforts to promote problem solving in school mathematics. It organized a two-part seminar 1986 and 1987 in problem solving for teacher educators. There were lectures and demonstrations, also in a school class, on the use of different problems. The participants were urged to apply these problems in their own teaching and to reflect upon them in the second part of the seminar a year later. In the seminar, the participants' conceptions of problem solving were charted with a questionnaire, and reported later on (Pehkonen 1993).

For example, the national curriculum for the comprehensive school (NBE 1994) provided rather general guidelines, and local schools were supposed to plan their more detailed curriculum documents within this framework (cf. Pehkonen & al. 2007). The importance of problem solving is clearly acknowledged in the curricular documents (NBE 1985, 1994, 2004).

Problems in textbooks. Before the problem-solving seminars in 1986–87, problem tasks were rather rare in Finnish mathematics textbooks. After the seminar almost every printing house published a set of problems, either as a booklet or as a deck of cards, and with time some problems were taken into the textbooks. But still in the beginning of the 1990's, a study shows that in the Finnish textbooks for grade 7 the proportion of problem tasks was about 11 % of all tasks (Kari 1991). Further non-systematic investigations of Finnish mathematics textbooks by teacher students show that the number of problem tasks has not essentially increased in the last decade.

The 1990's was a very fruitful decade in Finnish mathematics education. The National Board of Education published a guide book (Seppälä 1994) to help teachers when implementing the curricular framework (NBE 1994). Furthermore, new textbooks (usually three or four competing series) were elaborated and published according to the curricular framework. For that decade both in the elementary level (grades 1–6) and in upper level (grades 7–9) of the comprehensive school, there was published a book series that was devoted to train especially pupils' thinking and problem solving skills.

For example, the mathematics book for grades 7–9 of the comprehensive school "Matka matematiikkaan" [A Journey to Mathematics] (Espo & Rossi 1996) was launched. The focus of this textbook was teaching mathematics via problem solving, i.e. almost all contents were introduced via proper problem situations. On one side the use of the book demanded much

preparatory work from teachers' side, but on the other hand it made mathematics teaching more interesting and for pupils an adventure.

But the time seems not to be ripe for such radical textbooks, since teachers were not willing accept them but to stick in traditional mathematics books. For example, the upper school text book (Espo & Rossi 1996) was selected only by a few teachers, less than 10 %. But the influence of these books can be seen in the next generation of mathematics text books from other printing houses.

Use of problems in mathematics lessons. In the 1980's, there was much teacher in-service training for teachers of comprehensive school on activating teaching methods and problem solving. These components could be seen also in teachers' beliefs. Both elementary teachers and mathematics teachers regard problem solving as an important aspect of mathematics teaching. However, results after twenty years show that only some of the teachers have changed their teaching style. (Kupari, 1999) Even teachers who express beliefs favorable to problem solving, often fail to implement it in their own teaching. This phenomenon of unsuccessful teacher change has been dealt with in a recently published paper (Pehkonen 2006).

Although the development in problem solving has not been as rapid as expected, there are some changes to be observed. The use of problem solving tasks is quite popular today in Finnish mathematics lessons, but mainly in the form of mathematical puzzles. If we use the language introduced by Schroeder & Lester (1989), we might say that only few teachers are teaching *via* problem solving, while most of them teach something *about* problem solving. The latter means that they might use some mathematical puzzles in their teaching or have a problem box in their class or something similar. And the former states that these teachers use problem solving as a teaching method, and that is still very rare.

Using open-ended problems – a try for change

In the world-wide attempts to find a new teaching method that might meet the challenges set by constructivism, the so-called open approach was developed in the 1970's in Japan (e.g. Becker & Shimada 1997, Nohda 2000). Internationally it is accepted that open-ended problems form a useful tool in the development of mathematics teaching in schools, in a way that emphasizes understanding and creativity (e.g. Nohda 1991, Silver 1993, Stacey 1995).

In Finland, the ideas of open approach have been spread out in teacher in-service courses, in teachers' journals, and in teacher pre-service education for more than twenty years. The leading idea has been to increase openness and creativity in mathematics teaching. For example, the booklet Halinen & al. (1991) visioned the development in mathematics teaching for the 1990's including problem solving and open approach.

What are open problems? Tasks are said to be open, if their starting or goal situation is not exactly given (cf. Pehkonen 1995). Open is an opposite to closed, in the sense that a task is said to be closed, if the starting situation is exactly given, i.e. the task is well-defined, and if there is one certain result. In open tasks, pupils are given freedom, possibly even in the posing of the question, but at least in the solving of the task. In practice this means that they may end up with different, but equally correct solutions, depending on the additional choices made and the emphases placed during their solution processes. Therefore, open tasks usually have several correct answers.

When open tasks are used in mathematics teaching, pupils have an opportunity to act like creative mathematicians (cf. Brown 1997). Open problems encompass several types of problems (cf. Pehkonen 1995). Several examples of different types of open problems can be found e.g. in the papers by Nohda (1991, 2000), Silver (1993), Stacey (1995), and Pehkonen (2004).

Concluding notes

Summarizing the Finnish experiences of problem solving in mathematics education, we could state that teachers in Finland are changing in the direction of a more favorable attitude to problem solving. But its use in teaching demands much from the teacher, and, therefore, they find excuses why not to use a problem-solving approach. The younger generation of teachers seem to be more self-confident and open for changes. In a paper Pehkonen (2008), factors influencing changes in Finnish mathematics education of the last 30 years have been documented.

The positive experiences of the use of problem fields are similar to the ones reported e.g. by Liljedahl (2004). As part of a compulsory mathematics course he presented a group of pre-service elementary teachers a set of mathematical problems to solve. Some of the tasks allowed a form of mathematical discovery that he called a 'chain of discovery'. They facilitated a state of sustained engagement and even helped to change the student teachers' negative beliefs and attitudes.

References

Becker, J. & Shimada, L. 1997. *The open-ended approach*. Reston (VA): NCTM.

Brown, S. I. 1997. Thinking Like a Mathematician: A Problematic Perspective. *For the Learning of Mathematics* 17 (2), 36–38.

Espo, K. & Rossi, M. 1996. *Matka matematiikkaan – oppikirjasarja* [A journey to mathematics – a textbook series]. Helsinki: Edita.

Halinen, I., Hänninen, L., Joki, J., Leino, J., Näätänen, M., Pehkonen, E., Pehkonen, L., Sahlberg, P., Sainio, E., Seppälä, R. & Strang, T. (1991). *Peruskoulun matematiikan opetuksen kehityssuunnasta 1990-luvulla* [On the trend of mathematics teaching in the comprehensive school in the 1990's.]. Opetus & Kasvatus. Helsinki: VAPK-kustannus.

Kantowski, M.G. 1980. Some Thoughts on Teaching for Problem-Solving. In: *NCTM Yearbook 1980*, 195–203. Reston (VA): Council.

Kari, K. 1991. *Ongelmanratkaisu 1990-luvun alkupuolen 7. luokan matematiikan oppikirjoissa*. [Problem solving in the textbook of the grade in the beginning of the 1990's.] University of Helsinki. Department of mathematics (Master's thesis).

Kupari, P. 1999. *Laskutaitoharjoittelusta ongelmanratkaisuun. Matematiikan opettajien matematiikkauskomukset opetuksen muovaajina*. [From practising computational skills to problem solving. Mathematics teachers' mathematical beliefs and the construction of their teaching]. University of Jyväskylä. Institute for Educational Research. Research Reports 7.

Liljedahl, P. 2004. Sustained engagement: Preservice teachers' experience with a chain of discovery. In: *The fourth Conference of the European Society for Research in Mathematics Education in Spain 2004*. In proceedings of CERME 4.

NBE 1985. *Peruskoulun opetussuunnitelman perusteet.* [Basics for the curricula of the comprehensive school] Kouluhallitus. Helsinki: Valtion painatuskeskus.

NBE 1994. *Framework Curriculum for the Comprehensive School 1994*. National Board of Education: Helsinki: Valtion painatuskeskus.

NBE 2004. *Perusopetuksen opetussuunnitelman perusteet 2004*. [Core curriculum for comprehensive education 2004.] National Board of Education: Helsinki.

NCSM 1989. Essential Mathematics for the Twenty–first Century. *Mathematics Teacher* 82 (6), 470–474.

NCTM 2000. *Principles and Standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nohda, N. 1991. Paradigm of the ”open-approach” method in mathematics teaching: Focus on mathematical problem solving. *International Reviews on Mathematical Education* 23 (2), 32–37.

Nohda, N. 2000. Teaching by Open-Approach Method in Japanese Mathematics Classroom. In: *Proceedings of the PME-24 Conference* (eds. T. Nakahara & M. Koyama), Vol.1, 39–53. Hiroshima University (Japan).

Pehkonen, E. 1993. What are Finnish teacher educators' conceptions about the teaching of problem solving in mathematics? *European Journal for Teacher Education* 16 (3), 237–256.

Pehkonen, E. 1995. Introduction: Use of Open-ended Problems. *International Reviews on Mathematical Education* 27 (2), 55-57.

Pehkonen, E. 2004. State-of-the-Art in Problem Solving: Focus on Open Problems. In: *ProMath Jena 2003. Problem Solving in Mathematics Education* (eds. H. Rehlich & B. Zimmermann), 93–111. Hildesheim: Verlag Franzbecker.

Pehkonen, E. 2006. What Do We Know about Teacher Change in Mathematics? In: *Kunskapens och lärandets villkor. Festskrift tillägnad professor Ole Björkqvist* (eds. L. Häggblom, L. Burman & A-S. Röj-Lindberg), 77–87. Åbo Akademi, Pedagogiska fakulteten, Specialutgåva Nr 1/2006. Vasa.

Pehkonen, E. 2008. Some background factors for the Finnish PISA results in mathematics. Mediterranean Journal for Research in Mathematics Education, Vol. 7 (1).

Pehkonen, E., Ahtee, M. & Lavonen, J. (eds) 2007. *How Finns Learn Mathematics and Science*. Sense Publishers: Rotterdam/Taipai.

Schroeder, T.L. & Lester, F.K. 1989. Developing Understanding in Mathematics via Problem Solving. In: *NCTM Yearbook 1989* (ed. P.R. Trafton), 31-42. Reston (VA): Council.

Schoenfeld, A.H. 1985. *Mathematical problem solving*. Orlando (FL): Academic Press.

Seppälä, R. (ed.) 1994. *Matematiikka – taitoa ajatella. Yläaste ja lukio* [Mathematics – a skill to think. Lower and upper secondary school]. Suuntana oppimiskeskus 24. Opetushallitus.

Silver, E.A. 1993. On mathematical problem posing. In: *Proceedings of the seventeenth PME conference* (eds. I. Hirabayashi, N. Nohda, K. Shigematsu & F.-L. Lin). Vol. I, 66–85. University of Tsukuba, Tsukuba (Japan).

Stacey, K. 1995. The Challenges of Keeping Open Problem-Solving Open in School Mathematics. *International Reviews on Mathematical Education* 27 (2), 62–67.

The educational system of Finland is considered to be one of the best in the world. A high level of education is one of the courses of the Finnish national strategy. The main line of the education system is its democratic character: equal opportunities to get an education for every resident in Finland. That means that all foreigners of compulsory school age have the right to receive the same basic education as Finns. In Finland, the emphasis is put on the process of training, but not on the examinations.Â So what is the educational secret of the Finnish success? Education in Finland really gives universal skills; it is more than useful knowledge: it is an ability to use this knowledge in practice. It is that case when less is more: quality of processes defines their success. SHARE. Facebook. Twitter.